

# Topological Currents in Extended Electrodynamics, Negative Effective Dispersion, and Supersymmetric Consistency Constraints

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## Abstract

We investigate a proposed extension of four-dimensional electrodynamics in which a closed three-form current  $J_s \in \Omega^3(X)$  is constructed from higher-dimensional geometric data through fiber integration over a  $(4+n)$ -dimensional bulk bundle  $Y \xrightarrow{\pi} X$ . We distinguish two differential-form constructions: the pushforward of even-degree Chern–Weil forms built from the Chern character and the Dirac  $\hat{A}$ -genus (whose square equals the Todd class of the complexified vertical bundle,  $\text{Td}(T_\pi \otimes \mathbb{C}) = \hat{A}(T_\pi)^2$ ) over odd-dimensional real fibers ( $n = 2m - 1$ ), and the use of odd Chern-character data in  $K^{-1}(Y)$  over even-dimensional complex fibers ( $n = 2m$ ). For a globally defined gauge potential  $A \in \Omega^1(X)$  with field strength  $F = dA$ , the coupling  $A \wedge J_s$  defines a gauge-invariant action yielding the modified Maxwell equation  $d * F = J_s$  alongside  $dF = 0$ . Because  $*F$  is globally defined,  $d * F = J_s$  requires  $J_s$  to be globally exact ( $[J_s]_{\text{dR}} = 0$ ) on  $X$ , supporting non-zero charge on bounded spatial regions while having vanishing periods on closed three-cycles; we also outline how nontrivial bundles can be coupled via  $F \wedge B_s$  when  $J_s = dB_s$ , and identify defect and higher-gerbe extensions as directions requiring further construction if  $[J_s]_{\text{dR}} \neq 0$ .

We then examine how a dimension-six higher-spatial-derivative operator, defined relative to a preferred background direction  $u^\mu$  associated with  $*J_s$  and acting selectively on the spatially projected magnetic two-form  $F_{\mu\nu}^\perp$ , modifies photon dispersion while avoiding additional higher-time-derivative modes at the classical level. Within the sub-threshold momentum regime  $0 < q < q_{\text{stab}} = 1/\sqrt{\beta} = \Lambda^n/\sqrt{\alpha_s}$  (with  $\beta = \alpha_s/\Lambda^{2n}$  in natural units  $\hbar = c = 1$ ), the truncated quadratic model exhibits a negative dispersion curvature mass  $m^*(q) \simeq -1/(3\beta q) = -\Lambda^{2n}/(3\alpha_s q) < 0$  while maintaining a positive canonical quadratic Hamiltonian.

Finally, we discuss formal local electric-magnetic dualization, connections with the Kapustin–Witten framework, and supersymmetric operator protection. We formulate the supersymmetry connection as an open research conjecture regarding whether microscopic  $\mathcal{N} \geq 1$  BPS completions protect the spatially projected operator structure against radiative temporal-derivative mixing, and we derive conditional fixed-energy interferometric phase shifts and benchmark constraints in terms of the differential arm-to-arm coupling variance.

## 1 Introduction and Classification of Results

Recent ATLAS and CMS searches at the Large Hadron Collider (LHC) constrain gluino and squark production across a range of simplified supersymmetric models, with exclusion reaches depending on the assumed decay channels, branching ratios, and mass splittings. For example, a recent ATLAS analysis of events with  $\tau$ -leptons, jets, and missing transverse momentum reports gluino exclusions reaching up to 2.25 TeV and squark exclusions up to 1.7 TeV within specific simplified decay chains [1, 2]. While such searches increasingly constrain conventional low-energy supersymmetric scenarios, they do not establish universal lower mass limits across the full supersymmetric parameter space, nor do they diminish the role of supersymmetry (SUSY) as a theoretical framework in quantum field theory for investigating ultraviolet (UV) completeness, modular covariance, and non-perturbative operator protection.

In parallel, extensions of classical four-dimensional electrodynamics,

$$dF = 0, \quad d * F = 0, \quad (1)$$

have sought to incorporate hidden topological sectors and higher-dimensional couplings without violating precision low-energy electromagnetic tests. In an initial exploratory preprint [3], it was hypothesized that Maxwell’s equations could be extended via higher-dimensional sheaf interactions (termed “Photon Soul Continuity”). However, that preliminary formulation relied on inhomogeneous differential expressions and heuristic pushforwards that required formal mathematical refinement.

To maintain clear scientific boundaries in this revised manuscript, we explicitly separate our presentation into three distinct categories:

1. **Established Mathematical Derivations (Sections 2 and 3):** Calculations that follow deductively from stated geometric and variational assumptions. These include the construction of a closed 3-form  $J_s \in \Omega^3(X)$  via fiber integration in both odd and even fiber dimensions, the variational derivation of  $d * F = J_s$  for a globally defined gauge potential  $A \in \Omega^1(X)$ , the global exactness requirement  $[J_s]_{\text{dR}} = 0$  on  $X$ , and the derivation of the dispersion relation  $\omega(q)^2 = q^2(1 - \beta q^2)$ , its classical stability threshold  $q_{\text{stab}} = 1/\sqrt{\beta} = \Lambda^n/\sqrt{\alpha_s}$ , and its negative effective curvature mass  $m^*(q) \simeq -1/(3\beta q) < 0$  within a truncated quadratic spatial-gradient model.
2. **Proposed Theoretical Framework and Conditional Phenomenology (Sections 4.1 and 5):** The formal local path-integral dualization of the sourced action, the conditional retarded Green’s function expression for the holonomy phase on globally hyperbolic spacetimes, the fixed-energy inversion for the dispersive interferometric phase shift, and the conditional parameterization of fringe visibility loss in terms of the differential arm-to-arm coupling fluctuation variance  $\sigma_\eta^2$ .
3. **Conjectures and Open Problems (Sections 2.3, 4.2, and 6):** Research hypotheses and proposed extensions requiring further mathematical development, including the completion of a global higher-gerbe theory when  $[J_s]_{\text{dR}} \neq 0$ , and the open conjecture that microscopic  $\mathcal{N} \geq 1$  supersymmetric completions protect the spatially projected operator structure against radiative temporal-derivative mixing.

## 2 Cohomological Construction of the Topological 3-Form Current

### 2.1 Fiber Integration in Odd and Even Compactification Dimensions

Let  $(X, g)$  be an oriented, time-oriented, smooth 4-dimensional Lorentzian manifold with metric signature  $(-, +, +, +)$ . Let  $\pi : Y \rightarrow X$  be a smooth fiber bundle of total real dimension  $4 + n$  with compact, oriented, boundaryless fibers  $K = \pi^{-1}(x)$  ( $\partial K = \emptyset$ ) of characteristic length scale  $\Lambda^{-1}$ .

Integration along the fiber defines a linear pushforward map on smooth differential forms that lowers form degree by the real fiber dimension  $n$ :

$$\int_{Y/X} : \Omega^{3+n}(Y) \longrightarrow \Omega^3(X). \quad (2)$$

To construct a closed 3-form  $J_s \in \Omega^3(X)$  from higher-dimensional bundle data, we require a closed differential form  $\Theta \in \Omega^{3+n}(Y)$  ( $d_Y \Theta = 0$ ). Because standard characteristic forms reside in even degrees, the construction depends on the parity of the real fiber dimension  $n$ :

**Case I: Odd-Dimensional Real Fibers** ( $n = 2m - 1$ ,  $m \in \mathbb{Z}_{\geq 1}$ ). When the fiber  $K$  has odd real dimension (such as  $n = 1$  for  $K = S^1$  or  $n = 3$  for  $K = S^3$ ), the required bulk form degree is even:

$$3 + n = 3 + (2m - 1) = 2m + 2 \in 2\mathbb{Z}. \quad (3)$$

Let  $\mathcal{H} \rightarrow Y$  be a smooth complex vector bundle (the bulk Higgs bundle) equipped with a connection of curvature  $F_{\mathcal{H}}$ , and let  $T_{\pi} = \ker(d\pi) \subset TY$  denote the real vertical tangent bundle along the fibers, equipped with an induced Riemannian metric and Levi-Civita connection of curvature  $R_{\pi}$ . Because  $T_{\pi}$  is a real vector bundle of odd rank  $2m - 1$ , it does not carry a native complex structure. However, at the level of smooth Chern–Weil differential forms, its real Pontryagin forms  $p_k(T_{\pi}) \in \Omega^{4k}(Y)$  and its Dirac roof genus form  $\hat{A}(T_{\pi})$  are well-defined for any Riemannian connection on  $T_{\pi}$ :

$$\hat{A}(T_{\pi}) := \det^{1/2} \left( \frac{R_{\pi}/4\pi}{\sinh(R_{\pi}/4\pi)} \right) = 1 - \frac{1}{24}p_1(T_{\pi}) + \cdots \in \Omega^{4\bullet}(Y), \quad (4)$$

which is related to the Todd form of the complexified vertical bundle  $T_{\pi} \otimes \mathbb{C}$  by  $\text{Td}(T_{\pi} \otimes \mathbb{C}) = \hat{A}(T_{\pi})^2$ . We define the even-degree closed Chern–Weil representative on  $Y$  as:

$$\Theta_{\text{even}} := \kappa_n \left[ \text{Tr} \exp \left( \frac{iF_{\mathcal{H}}}{2\pi} \right) \wedge \hat{A}(T_{\pi}) \right]^{(2m+2)} \in \Omega^{2m+2}(Y), \quad (5)$$

where  $\kappa_n \in \mathbb{R} \setminus \{0\}$  is a normalization constant. By the Bianchi identity for  $F_{\mathcal{H}}$  and  $R_{\pi}$ ,  $\Theta_{\text{even}}$  is closed ( $d_Y \Theta_{\text{even}} = 0$ ).

**Case II: Even-Dimensional Complex Fibers** ( $n = 2m$ ,  $m \in \mathbb{Z}_{\geq 1}$ ). When the fiber  $K$  is a complex manifold of even real dimension  $n = 2m$  (such as a complex curve for  $n = 2$  or a K3/Calabi–Yau fiber for  $n = 4, 6$ ), the vertical tangent bundle  $T_{\pi}$  is a complex vector bundle of complex rank  $m$  with a well-defined Todd form  $\text{Td}(T_{\pi}) \in \Omega^{2\bullet}(Y)$ , and the required bulk degree is  $2m + 3$ , which is odd. Standard even-degree Chern–Weil polynomials cannot contribute to  $\Omega^3(X)$  under  $\int_{Y/X}$ . Instead, one supplies an odd  $K$ -theory class  $[\eta] \in K^{-1}(Y)$  [6], represented by a smooth map  $g : Y \rightarrow \text{U}(N)$ . Its odd Chern character has closed components in every odd degree  $2\ell + 1$ :

$$\text{ch}_{2\ell+1}(g) := (-1)^{\ell} \frac{\ell!}{(2\pi i)^{\ell+1} (2\ell+1)!} \text{Tr} \left[ (g^{-1} d_Y g)^{2\ell+1} \right] \in \Omega^{2\ell+1}(Y). \quad (6)$$

Taking the degree- $(2m + 3)$  component of the wedge product with  $\text{Td}(T_{\pi})$  defines the closed odd bulk form:

$$\Theta_{\text{odd}} := \kappa_n [\text{ch}_{\text{odd}}(g) \wedge \text{Td}(T_{\pi})]^{(2m+3)} \in \Omega^{2m+3}(Y). \quad (7)$$

**Proposition 2.1** (Closure and Pullback Vanishing of  $J_s$ ). *Let  $\Theta \in \Omega^{3+n}(Y)$  be a closed bulk form constructed via Case I ( $n$  odd) or Case II ( $n$  even), and define:*

$$J_s := \int_{Y/X} \Theta \in \Omega^3(X). \quad (8)$$

*Then:*

1.  $J_s$  is closed on  $X$ :  $dJ_s = 0$ .
2. If the bulk bundle data is pulled back trivially from  $X$  (so that  $\Theta = \pi^* \beta$  for some  $\beta \in \Omega^{3+n}(X)$  with  $n \geq 1$ ), then  $J_s \equiv 0$ .

*Proof.* For a smooth fiber bundle with compact, boundaryless fibers ( $\partial K = \emptyset$ ), fiber integration commutes with the exterior derivative:

$$dJ_s = d \left( \int_{Y/X} \Theta \right) = \int_{Y/X} d_Y \Theta = 0, \quad (9)$$

since  $d_Y \Theta = 0$ . Second, if  $\Theta = \pi^* \beta$ , then  $\Theta$  has zero vertical form degree along the  $n$ -dimensional fiber  $K$  (and  $\Omega^{3+n}(X) = 0$  on a 4-manifold whenever  $n > 1$ ), so the fiber integral vanishes identically.  $\square$

*Remark 2.2* (Chern–Weil Forms,  $K^1(X)$  Family Index Interpretation, and Rational Cohomology). Three mathematical distinctions govern the interpretation of  $\Theta_{\text{even}}$ :

1. *Differential forms vs.  $K^1(X)$  family Dirac index interpretation:* Proposition 2.1 requires only that  $T_\pi$  be an oriented Riemannian real vector bundle, which suffices to define  $\hat{A}(T_\pi)$  as a closed Chern–Weil differential form. If one wishes to interpret the fiber integral through the Atiyah–Singer family Dirac index theorem, the odd-dimensional spin fibers lead naturally to a  $K^1(X)$ -valued family index class (rather than an ordinary  $K^0(X)$  virtual bundle), whose odd Chern character is related to the fiber integral of  $\text{ch}(\mathcal{H}) \wedge \hat{A}(T_\pi)$  in rational cohomology, subject to  $T_\pi$  carrying a **spin structure** and standard index-theoretic normalization conventions. (If  $T_\pi$  instead carried a  $\text{Spin}^c$  structure with determinant line bundle  $L$ , the characteristic integrand would include an additional factor  $\exp(c_1(L)/2)$ , modifying  $\Theta_{\text{even}}$  to  $\text{ch}(\mathcal{H}) \wedge e^{c_1(L)/2} \wedge \hat{A}(T_\pi)$ .)
2. *Pointwise differential refinement:* Even when the cohomology classes match, a closed Chern–Weil fiber integral and the curvature representative of a differential  $K$ -theory refinement are not automatically identical pointwise as differential forms; a full differential index construction generally involves additional transgression or Bismut–Freed  $\eta$ -form data [6].
3. *Rational vs. integral periods:* Because the characteristic power series  $\text{ch}$ ,  $\hat{A}$ , and  $\text{Td}$  contain rational denominators, the real de Rham cohomology class  $[J_s]_{\text{dR}} \in H_{\text{dR}}^3(X; \mathbb{R})$  for  $\kappa_n = 1$  lies in the image of rational cohomology under the natural coefficient homomorphism  $H^3(X; \mathbb{Q}) \rightarrow H^3(X; \mathbb{R})$ , rather than automatically having integer periods. Achieving integral periods ( $\int_{\Sigma^3} J_s \in \mathbb{Z}$ ) requires choosing  $\kappa_n$  to clear the characteristic denominators on  $Y$  or imposing an integral differential-cohomological lift as an additional assumption.

## 2.2 Variational Principle in the Trivial-Bundle Sector and Global Coupling

To ensure that the action integral is globally well-defined on  $X$  without patching ambiguities, we formulate the primary variational principle for a **globally defined gauge potential**  $A \in \Omega^1(X)$  with field strength  $F = dA \in \Omega^2(X)$  (corresponding to a connection on a trivial principal  $U(1)$ -bundle over  $X$ ). Given a background closed 3-form  $J_s \in \Omega^3(X)$ , the wedge product  $A \wedge J_s \in \Omega^4(X)$  is then a globally defined 4-form on  $X$ , and we consider the action:

$$S[A] = \int_X \left( -\frac{1}{2} F \wedge *F + A \wedge J_s \right). \quad (10)$$

**Proposition 2.3** (Gauge Invariance and Euler–Lagrange Equations for Global  $A \in \Omega^1(X)$ ). *Let  $A \in \Omega^1(X)$  be a globally defined 1-form on  $X$  with  $F = dA$ , and assume  $dJ_s = 0$ . Then the action (10) is invariant under gauge transformations  $A \rightarrow A + d\chi$  for all compactly supported  $\chi \in C_c^\infty(X)$ , and its Euler–Lagrange equations are:*

$$dF = 0, \quad (11)$$

$$d * F = J_s. \quad (12)$$

Furthermore,  $F$  satisfies the exact 2-form wave equation:

$$\square F = d(*J_s), \quad (13)$$

where  $\square = d\delta + \delta d$  is the Hodge-de Rham wave operator on  $\Omega^2(X)$  and  $\delta = *d*$  is the codifferential.

*Proof.* Under  $A \rightarrow A + d\chi$  with  $\chi \in C_c^\infty(X)$ , the action shifts by:

$$\delta_\chi S = \int_X d\chi \wedge J_s = \int_X d(\chi J_s) - \int_X \chi dJ_s = 0, \quad (14)$$

by Stokes' theorem and  $dJ_s = 0$ . For an arbitrary variation  $\delta A \in \Omega_c^1(X)$ , using  $\delta F \wedge *F = F \wedge *\delta F$  and the graded Leibniz identity  $d(\delta A \wedge *F) = d(\delta A) \wedge *F - \delta A \wedge d*F$ , we obtain:

$$\delta S = \int_X (-d(\delta A) \wedge *F + \delta A \wedge J_s) = \int_X \delta A \wedge (-d*F + J_s) = 0, \quad (15)$$

yielding  $d*F = J_s$ . Acting on  $F$  with  $\square = d\delta + \delta d$  and using  $dF = 0$  gives  $\square F = d(*d*F) = d(*J_s)$ .  $\square$

*Remark 2.4* (Alternative Global Formulation for Nontrivial  $U(1)$ -Bundles). For a nontrivial principal  $U(1)$ -bundle  $P \rightarrow X$  (with  $c_1(P) \neq 0$ ), the curvature  $F \in \Omega^2(X)$  is globally defined on  $X$ , but the connection 1-form  $A$  exists only locally on open patches, so  $\int_X A \wedge J_s$  is not globally defined as written. However, because  $d*F = J_s$  requires  $J_s$  to be globally exact ( $J_s = dB_s$  for a globally defined smooth 2-form  $B_s \in \Omega^2(X)$ ; see Section 2.3), one can instead define the globally well-posed interaction term:

$$S_{\text{int}}[F] = \int_X F \wedge B_s. \quad (16)$$

Under local variations  $\delta F = d(\delta A)$  with compact support, integrating Equation (16) by parts gives  $\delta S_{\text{int}} = \int_X d(\delta A) \wedge B_s = \int_X \delta A \wedge dB_s = \int_X \delta A \wedge J_s$ , reproducing the exact same local field equation  $d*F = J_s$  without requiring a globally defined 1-form  $A$ .

### 2.3 Global Exactness Requirement and Proposed Extensions for $[J_s]_{\text{dR}} \neq 0$

Because the electromagnetic field strength  $F \in \Omega^2(X)$  and its Hodge dual  $*F \in \Omega^2(X)$  are globally defined smooth 2-forms on  $X$  for any  $U(1)$  bundle, the field equation  $d*F = J_s$  forces  $J_s$  to be the exterior derivative of a globally defined 2-form on  $X$ :

$$d*F = J_s \text{ globally on } X \implies [J_s]_{\text{dR}} = 0 \in H_{\text{dR}}^3(X; \mathbb{R}). \quad (17)$$

We therefore distinguish our primary globally exact baseline theory from proposed generalized extensions:

1. **Primary Baseline Theory: Globally Exact  $J_s$  ( $[J_s]_{\text{dR}} = 0$  on  $X$ ).** In our baseline theory, we assume as an explicit condition on the bulk bundle configuration that  $J_s$  is globally exact on  $X$ :  $J_s = dB_s$  for a globally defined smooth 2-form  $B_s \in \Omega^2(X)$  (this holds, for example, when the bulk characteristic form  $\Theta$  is exact on  $Y$  and admits a globally defined relative transgression primitive  $\Theta = d_Y \Omega_{\text{trans}}$ ). Under this assumption,  $d*F = J_s$  is globally consistent across  $X$ . By Stokes' theorem, the period of  $J_s$  over any closed 3-cycle  $\Sigma^3$  without boundary ( $\partial\Sigma^3 = \emptyset$ ) vanishes:

$$\int_{\Sigma^3} J_s = \int_{\Sigma^3} d*F = 0 \quad (\partial\Sigma^3 = \emptyset). \quad (18)$$

Crucially, however,  $J_s(x) \neq 0$  can be locally non-zero and supports a non-vanishing conserved charge inside any bounded 3-dimensional spatial domain  $\Omega_3 \subset \Sigma^3$  with closed 2-boundary  $\partial\Omega_3 \cong S^2$ :

$$Q_s(\Omega_3) := \int_{\Omega_3} J_s = \oint_{\partial\Omega_3} *F \neq 0. \quad (19)$$

All local wave, dispersion, and interferometric calculations in Sections 3 and 5 operate entirely within this globally smooth baseline theory.

2. **Proposed Directions for Non-Trivial Cohomology** ( $[J_s]_{\text{dR}} \neq 0$ ): If the fiber integral on  $Y$  produces a non-exact cohomology class  $[J_s]_{\text{dR}} \neq 0$  with non-zero periods over closed 3-cycles, the standard equation  $d * F = J_s$  cannot hold globally on  $X$ . Extending the framework to  $[J_s]_{\text{dR}} \neq 0$  is an open theoretical direction that would require modifying the global field theory, for example via:

- *Compensating Defect Currents*: Introducing a 1-dimensional defect locus  $\Gamma \subset X$  supporting a singular current  $J_{\text{def}}$  in the same cohomology class ( $[J_{\text{def}}]_{\text{dR}} = [J_s]_{\text{dR}}$ ) so that  $d * F = J_s - J_{\text{def}}$  has a globally exact source.
- *Higher-Gerbe / Stueckelberg Proposal*: Covering  $X$  with patches  $\{U_i\}$  where  $J_s|_{U_i} = dB_s^{(i)}$  ( $B_s^{(i)} - B_s^{(j)} = d\Lambda_{ij}$ ) and replacing  $*F$  by a patched 2-form field  $\mathcal{H}_2^{(i)}$  obeying  $d(\mathcal{H}_2^{(i)} - B_s^{(i)}) = 0$ . We emphasize that this equation alone does not yet constitute a complete modified electromagnetic theory: turning this proposal into a complete theory requires specifying the global action, the higher-gauge transformation laws, and the constitutive relation linking  $\mathcal{H}_2^{(i)}$  to the electromagnetic curvature  $F$ .

### 3 Spatially Projected Effective Action and Negative Dispersion Curvature

#### 3.1 Motivation: Higher-Derivative Operators and the Lightcone Pole

Before introducing our effective higher-derivative action, we note two mathematical features of higher-derivative extensions of electrodynamics:

1. **Lorentz-Invariant Four-Derivative Operators**: Adding a manifestly Lorentz-invariant dimension-six operator  $\frac{c}{4\Lambda^2}(\nabla_\lambda F_{\mu\nu})(\nabla^\lambda F^{\mu\nu})$  yields the momentum-space wave equation  $q^2(1 + \frac{c}{\Lambda^2}q^2)A_\mu^\text{T}(q) = 0$ . This factored equation leaves the massless lightcone dispersion branch  $\omega^2 = |\mathbf{q}|^2$  completely unshifted. Treating the truncated fourth-order operator as an exact theory produces a formal additional pole at  $q^2 = -\Lambda^2/c$ ; while such a cutoff-scale pole does not automatically establish a physical instability within the low-energy effective field theory (EFT) domain of validity (see Section 4.2), a Lorentz-invariant  $(\nabla_\lambda F_{\mu\nu})^2$  term fails to generate any low-momentum dispersion modification on the photon branch itself.
2. **Equal Electric-Magnetic Spatial Scaling**: Similarly, applying a purely spatial derivative operator equally to the full invariant  $F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}^2 - \mathbf{E}^2)$  multiplies both  $\omega^2|\mathbf{A}|^2$  and  $|\mathbf{q}|^2|\mathbf{A}|^2$  by the same spatial factor, which factors out without modifying  $\omega(q) = |\mathbf{q}|$ .

Consequently, to obtain the quartic dispersion relation  $\omega(q)^2 = q^2 - (\alpha_s/\Lambda^{2n})q^4$  on the physical photon branch from a local quadratic action without introducing higher time derivatives, the dimension-six operator must break boost invariance via the background current  $J_s$  and contain a **single spatial gradient** acting selectively on the **spatially projected magnetic 2-form**.

### 3.2 Spatial Magnetic Gradient Action and Dimensional Analysis

Assume that in a macroscopic region of spacetime, the dual 1-form  $v := *J_s \in \Omega^1(X)$  defines a non-vanishing timelike background vector field  $v^\mu = v_0 u^\mu$ , where  $u^\mu$  is a dimensionless unit timelike four-velocity ( $g_{\mu\nu} u^\mu u^\nu = -1$ ). Define the orthogonal spatial projector:

$$P_\nu^\mu := \delta_\nu^\mu + u^\mu u_\nu, \quad P_\nu^\mu u^\nu = 0, \quad P_\alpha^\mu P_\nu^\alpha = P_\nu^\mu. \quad (20)$$

Relative to  $u^\mu$ , the field strength tensor  $F_{\mu\nu}$  decomposes orthogonally into an electric 1-form  $E_\mu$  and a purely spatial magnetic 2-form  $F_{\mu\nu}^\perp$ :

$$E_\mu := F_{\mu\nu} u^\nu, \quad F_{\mu\nu}^\perp := P_\mu^\alpha P_\nu^\beta F_{\alpha\beta}, \quad -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} E_\mu E^\mu - \frac{1}{4} F_{\mu\nu}^\perp F_{\perp}^{\mu\nu}. \quad (21)$$

We postulate the effective higher-derivative action built from the projected spatial gradient  $\nabla_\alpha^\perp := P_\alpha^\beta \nabla_\beta$  of  $F_{\mu\nu}^\perp$ :

$$S_{\text{eff}}[A] = \int_X d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu v^\mu + \frac{\alpha_s}{4\Lambda^{2n}} P^{\alpha\beta} (\nabla_\alpha F_{\mu\nu}^\perp) (\nabla_\beta F_{\perp}^{\mu\nu}) \right]. \quad (22)$$

Throughout this section, we work in 4D natural units ( $\hbar = c = 1$ ), in which the mass dimensions of the fields and operators are:

$$[A_\mu] = M^1, \quad [F_{\mu\nu}^\perp] = M^2, \quad [\nabla_\alpha] = M^1, \quad [P^{\alpha\beta}] = M^0 \implies [P^{\alpha\beta} (\nabla_\alpha F_{\mu\nu}^\perp) (\nabla_\beta F_{\perp}^{\mu\nu})] = M^6. \quad (23)$$

For the Lagrangian density to have mass dimension  $[\mathcal{L}] = M^4$ , the composite parameter

$$\beta := \frac{\alpha_s}{\Lambda^{2n}} = \frac{\tilde{\omega}_\eta}{\Lambda^2} \quad (24)$$

must have mass dimension  $[\beta] = M^{-2}$ , which fixes  $[\alpha_s] = M^{2n-2}$  and defines the dimensionless coupling  $\tilde{\omega}_\eta := \alpha_s \Lambda^{2-2n}$  ( $[\tilde{\omega}_\eta] = M^0$ ).

**Theorem 3.1** (Dispersion Relation, Stability Threshold, and Negative Curvature Mass of the Truncated Quadratic Model). *Let  $X = \mathbb{R}^{1,3}$  be Minkowski spacetime in natural units ( $\hbar = c = 1$ ) and work in the rest frame of the background vector  $u^\mu = (1, \mathbf{0})$  with constant  $v^\mu$ . For linear plane-wave perturbations  $\delta A_\mu(t, \mathbf{x}) \propto e^{-i\omega t + i\mathbf{q} \cdot \mathbf{x}}$  in radiation gauge ( $\delta A_0 = 0$ ,  $\nabla \cdot \delta \mathbf{A} = 0$ ) around the background solution, the effective action (22) with  $\alpha_s > 0$  satisfies:*

1. **Second-Order Time Derivatives and Fourth-Order Spatial Derivatives:** Up to a total spatial divergence, the quadratic perturbation Lagrangian is:

$$\mathcal{L}^{(2)} = \frac{1}{2} |\partial_t \delta \mathbf{A}|^2 - \frac{1}{2} (\partial_j \delta A_i) (\partial^j \delta A^i) + \frac{\alpha_s}{2\Lambda^{2n}} (\nabla^2 \delta A_i) (\nabla^2 \delta A^i). \quad (25)$$

2. **Dispersion Relation of the Truncated Quadratic Model:** Varying Equation (25) with respect to  $\delta A_i$  yields the fourth-order spatial wave equation:

$$\left( -\partial_t^2 + \nabla^2 + \frac{\alpha_s}{\Lambda^{2n}} \nabla^4 \right) \delta A_i = 0, \quad (26)$$

whose transverse Fourier modes ( $q \equiv |\mathbf{q}|$ ) obey the dispersion relation of the truncated quadratic model:

$$\omega(q)^2 = q^2 (1 - \beta q^2) = q^2 - \frac{\alpha_s}{\Lambda^{2n}} q^4. \quad (27)$$

3. **Dimensionally Consistent Classical Stability Threshold:** Define the momentum stability threshold of the quadratic model:

$$q_{\text{stab}} := \frac{1}{\sqrt{\beta}} = \frac{\Lambda^n}{\sqrt{\alpha_s}} = \frac{\Lambda}{\sqrt{\omega_\eta}}, \quad [q_{\text{stab}}] = M^1. \quad (28)$$

For all wavenumbers in the sub-threshold regime  $0 < q < q_{\text{stab}}$ , the frequency  $\omega(q)$  is real and positive, and the canonical quadratic Hamiltonian density of each mode  $\mathbf{q}$  is strictly positive. (We distinguish  $q_{\text{stab}}$ , the classical stability threshold of the truncated quadratic model, from the EFT cutoff  $\Lambda_{\text{EFT}} \sim \Lambda$ ; if  $q_{\text{stab}} \ll \Lambda$ , the dimension-six operator becomes large below  $\Lambda$ , so relying on the truncated action up to  $q_{\text{stab}}$  requires assuming that higher-order spatial operators remain parametrically suppressed.)

4. **Negative Effective Curvature Mass in Natural Units:** Defining the effective curvature mass in natural units ( $\hbar = c = 1$ ) by  $1/m^*(q) := d^2\omega(q)/dq^2$ , the low-momentum expansion for  $q \ll q_{\text{stab}}$  yields:

$$m^*(q) = -\frac{1}{3\beta q} \left[ 1 + \mathcal{O}\left(\frac{q^2}{q_{\text{stab}}^2}\right) \right] = -\frac{\Lambda^{2n}}{3\alpha_s q} \left[ 1 + \mathcal{O}\left(\frac{q^2}{q_{\text{stab}}^2}\right) \right] < 0, \quad [m^*] = M^1. \quad (29)$$

(Restoring physical SI units via  $\omega_{\text{SI}}^2(q) = c^2 q^2 (1 - \beta q^2)$  and  $1/m_{\text{SI}}^*(q) := \hbar^{-1}(d^2\omega_{\text{SI}}/dq^2)$  gives  $m_{\text{SI}}^*(q) \simeq -\hbar/(3c\beta q)$ .)

*Proof.* In the rest frame  $u^\mu = (1, \mathbf{0})$ , we have  $P^{0\mu} = 0$  and  $P^{ij} = \delta^{ij}$ . Thus  $E_i = F_{i0} = \partial_i A_0 - \partial_0 A_i$  and  $F_{ij}^\perp = F_{ij} = \partial_i A_j - \partial_j A_i$ , while  $F_{0\mu}^\perp = 0$ . In radiation gauge ( $\delta A_0 = 0$ ,  $\partial_i \delta A^i = 0$ ), the higher-derivative term in Equation (22) becomes:

$$\frac{\alpha_s}{4\Lambda^{2n}} \delta^{kl} (\partial_k F_{ij}) (\partial_l F^{ij}) = \frac{\alpha_s}{2\Lambda^{2n}} \left[ (\partial_k \partial_i \delta A_j) (\partial^k \partial^i \delta A^j) - (\partial_k \partial_i \delta A_j) (\partial^k \partial^j \delta A^i) \right]. \quad (30)$$

Integrating by parts twice spatially and using  $\partial_i \delta A^i = 0$ , the second term vanishes and the first term gives  $+\frac{\alpha_s}{2\Lambda^{2n}} (\nabla^2 \delta A_j) (\nabla^2 \delta A^j)$ , establishing Equation (25).

Varying  $\int dt d^3x \mathcal{L}^{(2)}$  with respect to  $\delta A_i$  gives  $(-\partial_t^2 + \nabla^2 + \beta \nabla^4) \delta A_i = 0$ . Substituting  $\partial_t \rightarrow -i\omega$  and  $\nabla^2 \rightarrow -q^2$  ( $\nabla^4 \rightarrow +q^4$ ) yields Equation (27). Because  $\mathcal{L}^{(2)}$  depends on  $\partial_t \delta \mathbf{A}$  solely through  $\frac{1}{2} |\partial_t \delta \mathbf{A}|^2$ , the canonical momentum is  $\mathbf{\Pi} = \partial_t \delta \mathbf{A}$ , and the canonical Hamiltonian density in Fourier space is:

$$\mathcal{H}_{\text{can}}(\mathbf{q}) = \frac{1}{2} |\mathbf{\Pi}(\mathbf{q})|^2 + \frac{1}{2} q^2 \left( 1 - \frac{q^2}{q_{\text{stab}}^2} \right) |\delta \tilde{\mathbf{A}}(\mathbf{q})|^2 > 0 \quad \forall q \in (0, q_{\text{stab}}). \quad (31)$$

Finally, expanding  $\omega(q) = q \sqrt{1 - (q/q_{\text{stab}})^2}$  for  $q \ll q_{\text{stab}}$  with dimensionally homogeneous remainder terms gives:

$$\omega(q) = q - \frac{q^3}{2q_{\text{stab}}^2} - \frac{q^5}{8q_{\text{stab}}^4} + \mathcal{O}\left(\frac{q^7}{q_{\text{stab}}^6}\right). \quad (32)$$

Differentiating twice with respect to  $q$  yields:

$$v_g(q) = \frac{d\omega}{dq} = 1 - \frac{3q^2}{2q_{\text{stab}}^2} + \mathcal{O}\left(\frac{q^4}{q_{\text{stab}}^4}\right), \quad \frac{d^2\omega}{dq^2} = -\frac{3q}{q_{\text{stab}}^2} + \mathcal{O}\left(\frac{q^3}{q_{\text{stab}}^4}\right) = -3\beta q + \mathcal{O}\left(\frac{q^3}{q_{\text{stab}}^4}\right). \quad (33)$$

Inverting  $d^2\omega/dq^2$  in natural units yields Equation (29).  $\square$

## 4 Formal Local Dualization and the Supersymmetric Protection Conjecture

### 4.1 Formal Path-Integral Dualization in the Globally Exact Theory

To examine how the topological source  $J_s = dB_s$  behaves under electric-magnetic duality within the globally exact baseline theory of Section 2.3, we perform a formal path-integral dualization [4]. Starting from the globally defined interaction  $\int_X F \wedge B_s$  of Remark 2.4, we promote  $F \in \Omega^2(X)$  to an independent 2-form field and enforce the Bianchi identity  $dF = 0$  via a Lagrange multiplier 1-form  $A_D$  with  $F_D = dA_D$ :

$$S_{\text{master}}[F, A_D] = \int_X \left( -\frac{1}{2g^2} F \wedge *F + \frac{\theta}{8\pi^2} F \wedge F + F \wedge B_s + \frac{1}{2\pi} F \wedge dA_D \right). \quad (34)$$

Performing the Gaussian integral over  $F$  replaces  $F_D$  in the dual action with the shifted 2-form combination:

$$\mathcal{F}_D := F_D + 2\pi B_s, \quad d\mathcal{F}_D = d(F_D + 2\pi B_s) = 2\pi dB_s = 2\pi J_s. \quad (35)$$

We note an important qualification regarding this identity: for an ordinary dual  $U(1)$  connection  $A_D$ , the curvature  $F_D = dA_D$  remains closed ( $dF_D = 0$ ), and the shifted combination  $\mathcal{F}_D = F_D + 2\pi B_s$  is not automatically the curvature of an ordinary  $U(1)$  connection. Thus,  $d(F_D + 2\pi B_s) = 2\pi J_s$  represents a formal local shift of the dual 2-form field strength by the smooth background potential  $B_s$ , rather than establishing  $J_s$  as a conventional global magnetic monopole current. Promoting  $J_s$  to a genuine cohomological magnetic monopole current in the dual frame would require completing the higher-gerbe extension outlined in Section 2.3, where  $B_s^{(i)}$  carries non-trivial Čech transitions and  $A_D$  transforms as a 1-gerbe trivialization.

### 4.2 EFT Qualification and the Supersymmetric Protection Conjecture

While Theorem 3.1 proves that the spatially projected action (22) avoids higher time derivatives at tree level in the condensate rest frame, quantum loop corrections coupling  $A_\mu$  to generic matter or gravitational fields can radiatively generate Lorentz-invariant four-derivative operators of the form:

$$\Delta\mathcal{L}_{\text{loop}} = c_{\text{loop}} \frac{e^2}{\Lambda^2} (\nabla_\lambda F_{\mu\nu})(\nabla^\lambda F^{\mu\nu}) \supset c_{\text{loop}} \frac{e^2}{\Lambda^2} (\partial_t^2 A_i)^2. \quad (36)$$

In a standard perturbative effective field theory valid strictly at  $q \ll \Lambda$ , a higher-time-derivative term suppressed by  $\Lambda^{-2}$  does *not* automatically represent a physical Ostrogradsky instability within the low-energy regime: the formal additional pole induced by treating the truncated Lagrangian as an exact theory lies at  $|q^2| \sim \Lambda^2/(c_{\text{loop}}e^2)$  (at or above the EFT cutoff), and at leading perturbative order the redundant operator  $(\nabla_\lambda F_{\mu\nu})(\nabla^\lambda F^{\mu\nu})$  can be removed via a local field redefinition  $A_\mu \rightarrow A_\mu + \delta A_\mu$  proportional to the lowest-order equations of motion.

However, a non-trivial consistency question arises if one investigates whether the spatially projected operator structure of Equation (22) can remain controlled without radiative mixing into higher time derivatives beyond leading perturbative truncation (for instance, in a UV-complete bulk theory, or in a regime with  $q_{\text{stab}} \ll \Lambda$  under the additional assumption that higher-dimension spatial operators remain suppressed so that the effective description stays controlled near  $q_{\text{stab}}$ ).

**Conjecture 4.1** (Supersymmetric Operator Protection Hypothesis). Let  $Y \xrightarrow{\pi} X$  be a microscopic bulk compactification whose low-energy effective theory reproduces the spatially projected action (22). We conjecture that if the bulk manifold  $Y$  admits a covariantly constant spinor  $\nabla_M \epsilon = 0$  and the background bundle  $\mathcal{H}$  satisfies a first-order Bogomolny–Prasad–Sommerfield (BPS) condition preserving a residual supersymmetry algebra aligned with the background frame  $u^\mu$ , then supersymmetric non-renormalization theorems protect the spatially projected kinetic structure against radiative mixing with fourth-order time-derivative counterterms.

*Remark 4.2* (Epistemological Status of Conjecture 4.1). Conjecture 4.1 is an open *research problem*, not a proven theorem of this manuscript. The calculations in Section 3 establish only the classical behavior of the effective model; determining whether a residual supersymmetry algebra is necessary or sufficient to forbid temporal-derivative mixing at loop level requires an explicit one-loop polarization tensor calculation in a concrete supersymmetric Kaluza–Klein model.

## 5 Conditional Phenomenological Signatures in Interferometry

### 5.1 Conditional Retarded Green’s Function Solution for the Holonomy Phase

To construct a particular solution of the exact 2-form wave equation (13),  $\square F = d(*J_s)$ , we assume in this section that the spacetime  $(X, g)$  is **globally hyperbolic**, so that the Hodge–de Rham wave operator  $\square = d\delta + \delta d$  on  $\Omega^2(X)$  admits a unique, globally defined retarded Green’s operator  $\square_{\text{ret}}^{-1}$ . Because the second-order wave equation  $\square F = d(*J_s)$  is a necessary consequence of the first-order Maxwell system  $dF = 0$  and  $d * F = J_s$  but does not by itself enforce the first-order constraints for arbitrary initial data, we assume that the source  $J_s$  has compact support in the causal past (or appropriate falloff) and that the Cauchy initial data and homogeneous solution  $F^{(0)} \in \Omega^2(X)$  (satisfying  $dF^{(0)} = 0$  and  $d * F^{(0)} = 0$ ) are chosen consistently so that the full retarded solution,

$$F(x) = F^{(0)}(x) + [\square_{\text{ret}}^{-1} d(*J_s)](x) \in \Omega^2(X), \quad (37)$$

satisfies the complete first-order Maxwell system  $dF = 0$  and  $d * F = J_s$ .

For a two-path interferometer whose arms  $\gamma_1, \gamma_2$  form the closed boundary  $\partial\Sigma^2 = \gamma_1 - \gamma_2$  of a 2-dimensional spatial surface  $\Sigma^2$ , Stokes’ theorem gives the conditional gauge-invariant holonomy phase shift of this Maxwell solution:

$$\Delta\Phi_{\text{soul}} = \oint_{\gamma_1 - \gamma_2} (A - A^{(0)}) = \int_{\Sigma^2} (F - F^{(0)}) = \int_{\Sigma^2} \square_{\text{ret}}^{-1} d(*J_s). \quad (38)$$

### 5.2 Fixed-Energy Phase Shift and Differential Arm-to-Arm Variance

In a stationary optical interferometer, a monochromatic beam is prepared at a fixed photon energy  $E_\gamma = \hbar\omega$ . Working in natural units ( $\hbar = c = 1$ , where  $\omega = E_\gamma$  and the unmodified vacuum wavenumber is  $q_0(E_\gamma) = E_\gamma$ ), we invert the dispersion relation (27),

$$E_\gamma^2 = \omega^2 = q^2 \left( 1 - \frac{\tilde{\omega}_\eta}{\Lambda^2} q^2 \right), \quad (39)$$

to find the modified propagation wavenumber  $q(E_\gamma)$  at fixed photon energy  $E_\gamma \ll \Lambda/\sqrt{\tilde{\omega}_\eta}$ :

$$q(E_\gamma) = E_\gamma \left( 1 + \frac{\tilde{\omega}_\eta}{2\Lambda^2} E_\gamma^2 + \mathcal{O} \left( \frac{\tilde{\omega}_\eta^2 E_\gamma^4}{\Lambda^4} \right) \right). \quad (40)$$

Along a single arm of length  $L$ , the dispersive phase shift accumulated at fixed photon energy  $E_\gamma$  relative to vacuum propagation is:

$$\Delta\Phi_{\text{disp}}(E_\gamma) = (q(E_\gamma) - q_0(E_\gamma))L = \frac{\tilde{\omega}_\eta}{2\Lambda^2} E_\gamma^3 L \left[ 1 + \mathcal{O} \left( \frac{\tilde{\omega}_\eta E_\gamma^2}{\Lambda^2} \right) \right]. \quad (41)$$

Now consider a two-arm interferometer in which the dimensionless effective couplings along arm 1 and arm 2 experience stochastic fluctuations  $\delta\tilde{\omega}_1$  and  $\delta\tilde{\omega}_2$ , respectively. The fluctuating phase difference between the two arms is:

$$\delta\Delta\Phi_{\text{disp}}(E_\gamma) = \frac{E_\gamma^3 L}{2\Lambda^2} (\delta\tilde{\omega}_1 - \delta\tilde{\omega}_2). \quad (42)$$

We define  $\sigma_\eta^2$  directly as the **differential arm-to-arm coupling variance**:

$$\sigma_\eta^2 := \text{Var}(\delta\tilde{\omega}_1 - \delta\tilde{\omega}_2) = \left\langle (\delta\tilde{\omega}_1 - \delta\tilde{\omega}_2)^2 \right\rangle. \quad (43)$$

(Note that if each individual arm experiences statistically independent, identically distributed fluctuations with single-arm variance  $\sigma_{\text{arm}}^2 := \text{Var}(\delta\tilde{\omega}_1) = \text{Var}(\delta\tilde{\omega}_2)$ , then  $\sigma_\eta^2 = 2\sigma_{\text{arm}}^2$ .) With the definition (43), the variance of the interferometric phase difference is:

$$\text{Var}(\delta\Delta\Phi_{\text{disp}}(E_\gamma)) = \frac{\sigma_\eta^2 E_\gamma^6 L^2}{4 \Lambda^4}, \quad (44)$$

and Gaussian ensemble averaging suppresses the interferometric fringe visibility  $\mathcal{V} := (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}})$  according to:

$$1 - \mathcal{V} \approx \frac{1}{2} \text{Var}(\delta\Delta\Phi_{\text{disp}}(E_\gamma)) = \frac{\sigma_\eta^2 E_\gamma^6 L^2}{8 \Lambda^4}. \quad (45)$$

Inverting Equation (45) (converting  $L$  from meters to  $\text{eV}^{-1}$  via  $L/(\hbar c)$  with  $\hbar c = 1.97327 \times 10^{-7} \text{ eV} \cdot \text{m}$ ) yields the conditional lower bound on  $\Lambda$ :

$$\Lambda \geq \left( \frac{\sigma_\eta^2 E_\gamma^6 (L/\hbar c)^2}{8(1 - \mathcal{V}_{\text{bound}})} \right)^{1/4}. \quad (46)$$

| Benchmark Scenario             | Photon Energy $E_\gamma$ | Effective Path $L$          | Assumed $1 - \mathcal{V}_{\text{bound}}$ | Conditional Bound               |
|--------------------------------|--------------------------|-----------------------------|------------------------------------------|---------------------------------|
| Optical Mach–Zehnder Benchmark | 1.50 eV                  | 10 m                        | $1.0 \times 10^{-9}$                     | $\Lambda \geq 1.38 \text{ MeV}$ |
| Fabry–Pérot Cavity Benchmark   | 1.17 eV                  | $1.0 \times 10^5 \text{ m}$ | $1.0 \times 10^{-10}$                    | $\Lambda \geq 169 \text{ MeV}$  |
| X-Ray Interferometry Benchmark | 10.0 keV                 | 100 m                       | $1.0 \times 10^{-6}$                     | $\Lambda \geq 423 \text{ GeV}$  |

Table 1: Conditional benchmark constraints on the scale  $\Lambda$  evaluated from Equation (46) at fixed photon energy  $E_\gamma$  for unit differential arm-to-arm variance ( $\sigma_\eta = 1$ , corresponding to single-arm variance  $\sigma_{\text{arm}} = 1/\sqrt{2}$  for independent arms). These entries represent parametric sensitivity benchmarks of the model rather than established experimental exclusions.

## 6 Limitations and Open Problems

To distinguish the derived mathematical results of this manuscript from open physical questions, we highlight five calculations required to complete the framework:

1. **Microscopic Kaluza–Klein Matching of  $\alpha_s$  and  $\kappa_n$ :** While Section 2 constructs the classical 3-form source  $J_s$  via fiber integration and Section 3 derives the dispersion relation of the truncated quadratic model (22), the Wilson coefficient  $\alpha_s$  is treated as an independent effective parameter. Deriving  $\alpha_s$  from first principles requires integrating out heavy Kaluza–Klein modes of  $\mathcal{H}$  at one loop on a specified bulk geometry  $Y = X \times K$  with non-zero background flux  $\Theta$ .
2. **Completion of the Global Higher-Gerbe Theory for  $[J_s]_{\text{dR}} \neq 0$ :** While the baseline theory is globally consistent for  $[J_s]_{\text{dR}} = 0$ , extending the framework to non-trivial cohomology classes  $[J_s]_{\text{dR}} \neq 0$  requires constructing a complete global higher-gauge action, specifying the gauge transformations of the patched 2-form  $\mathcal{H}_2^{(i)}$ , and defining its exact relation to the electromagnetic curvature  $F$ .

3. **Dynamical Stability of the Preferred Frame Vector  $u^\mu$ :** In Section 3, the unit timelike vector  $u^\mu$  aligned with  $*J_s$  was treated as a fixed background field. In a fully dynamical theory where  $J_s$  fluctuates, the kinetic terms and constraint structure of  $u^\mu$  (analogous to Einstein–Aether vector fields) must be analyzed to verify that longitudinal modes remain ghost-free.
4. **Explicit One-Loop Test of Conjecture 4.1:** Determining whether residual  $\mathcal{N} \geq 1$  supersymmetry prevents radiative mixing between spatial gradients  $P^{\alpha\beta}\nabla_\alpha\nabla_\beta$  and time derivatives  $(u^\alpha\nabla_\alpha)^2$  requires an explicit one-loop polarization tensor calculation in a supersymmetric bulk completion.
5. **Microscopic Noise Modeling for  $\sigma_\eta^2$ :** Converting the conditional benchmark sensitivities in Table 1 into actual experimental bounds requires a microscopic quantum-optical model predicting the differential arm-to-arm fluctuation variance  $\sigma_\eta^2$  and its spatial correlation length relative to the interferometer baseline.

## 7 Conclusion

We have developed a degree-consistent mathematical framework extending four-dimensional electrodynamics via a closed topological 3-form current  $J_s \in \Omega^3(X)$  constructed by fiber integration over odd- or even-dimensional compact fibers, using closed Chern–Weil forms built from the Dirac  $\hat{A}$ -genus on odd real fibers and odd  $K$ -theory Chern characters on even complex fibers. For a globally defined gauge potential  $A \in \Omega^1(X)$ , we derived the sourced Maxwell equation  $d * F = J_s$  and the exact 2-form wave equation  $\square F = d(*J_s)$ . Because  $*F$  is globally defined, we showed that  $d * F = J_s$  requires  $[J_s]_{\text{dR}} = 0$  on  $X$  in the globally smooth baseline theory (supporting non-zero charge on bounded spatial regions), while also showing how nontrivial  $U(1)$ -bundles can be coupled via  $\int_X F \wedge B_s$  when  $J_s = dB_s$  and identifying defect and higher-gerbe formulations as open directions for  $[J_s]_{\text{dR}} \neq 0$ .

Furthermore, we demonstrated that a dimension-six operator built from the projected spatial gradient of the magnetic 2-form  $F_{\mu\nu}^\perp$  generates the modified dispersion relation  $\omega(q)^2 = q^2(1 - \beta q^2)$  within the truncated quadratic model in natural units ( $\hbar = c = 1$ ), with a dimensionally consistent stability threshold  $q_{\text{stab}} = 1/\sqrt{\beta} = \Lambda^n/\sqrt{\alpha_s}$  and a negative effective curvature mass  $m^*(q) \simeq -1/(3\beta q) = -\Lambda^{2n}/(3\alpha_s q) < 0$ , while keeping the classical quadratic Lagrangian strictly second-order in time derivatives. While these results establish the classical behavior of the proposed effective model, whether a microscopic  $\mathcal{N} \geq 1$  supersymmetric completion protects this spatially projected operator structure against radiative temporal-derivative mixing at loop level remains an open research conjecture for future investigation.

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