An Open Letter to the CMB Cosmology, Relativistic-Gravity, and Holographic-Theory Communities: Gauge-Invariant Torsional Anomaly Inflow, Kaluza–Klein Soliton Obstructions, Nonsingular Bounce Bounds, and Black Hole Soft-Hair Consistency

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*Gravity, Holography & Cosmology Working Note: Exact Index-Theoretic Inflow, Corrected Radion-Soliton Systems, and No-Go Theorems,
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Abstract

This document is written as an open letter to researchers in observational cosmology, relativistic gravitation, Kaluza–Klein field theory, and holographic quantum gravity. Across seventeen recent preprints [1–17], we explored torsional anomaly inflow in warped compactifications, black hole electromagnetic soft hair, radion-supported gravitating solitons, $f(R)$ nonsingular cosmological bounces, and ultrahyperbolic $(3+3)$ spacetime geometry. A systematic re-evaluation reveals sharp geometric, variational, and observational obstructions across each of those sectors.

Here we present four rigorous resolutions. First, replacing naive five-dimensional Atiyah–Patodi–Singer formulas with Dai–Freed anomaly inflow, restoring the $[M]^{-2}$ scaling of the Nieh–Yan invariant, fixing Pomarol's warped gauge-mode ODE, and correcting a factor-of-two CMB birefringence coefficient bounds cosmological axial torsion at $|S_0| \lesssim 2\times10^{-42}\,\mathrm{GeV}$, eleven orders of magnitude tighter than terrestrial spin-pendulum limits. Second, integrating Kerr–Newman soft hair via Haco–Perry–Strominger preserves exact classical $S_{\rm BH} = A_H/(4G)$ while isolating Sen's one-loop logarithmic shifts and Kontsevich–Segal–Witten saddle allowability. Third, we publish the corrected five-ODE Einstein–Yang–Mills–Higgs–radion system with an exact Minkowski vacuum and prove a no-go theorem forbidding globally regular static Einstein–radion–Born–Infeld electric solitons. Fourth, we prove an exact no-bounce theorem for ghost-free flat-FLRW $R+\alpha R^2$ gravity ($\alpha>0$) and specify the causal and empirical obstructions to $(3+3)$ spacetime.

What follows is a technical statement of these four constructive resolutions and exact no-go theorems, together with a direct request that cosmological and gravitational groups test the viable parameter windows. The discussion is organized in six points.

(1) Gauge-invariant torsional anomaly inflow, warped localization, and CMB birefringence bounds. In [1–5], several mathematical obstructions hindered the torsional-photon program: (i) minimal covariantization $\nabla_\mu A_\nu - \nabla_\nu A_\mu$ fails $U(1)$ gauge invariance ($\delta F_{\mu\nu} = -T^\lambda{}_{\mu\nu}\partial_\lambda\lambda \neq 0$), whereas $F = dA$ is torsion-free; (ii) the Nieh–Yan 4-form $N_{\rm NY} = T^a \wedge T_a - R_{ab}\wedge e^a \wedge e^b = d(e^a \wedge T_a)$ carries mass dimension $[M]^{-2}$ (not dimensionless), requiring a physical cutoff $M_{\rm UV}^2$ or thermal $T^2/12$ coefficient [18]; (iii) the bulk–boundary Atiyah–Patodi–Singer theorem requires *even* bulk and *odd* boundary dimension, whereas [1–3] paired 4D even spacetime $X_4 = \partial Y$ with compact Kähler fibers ($\partial Y = \emptyset$); (iv) [1] modified $dF = *J_{\rm anom}$ while [2, 3] set $d(*F - J_{\rm top}) = 0$ (a 2-form minus a 3-form) and [1] Eq. (5.1) gave $V \to 0$ as $J_{\rm top}\to 0$; and (v) in 5D Randall–Sundrum [4], the Hodge dual of antisymmetric torsion T_{PQR} is a 2-form $S_{MN} = \frac{1}{6}\epsilon_{MNPQR}T^{PQR}$ (not a 4-index dual), $e^{-2k\pi R}\sim 10^{-32}$ suppresses rather than enhances IR boundary effects, and Eq. (9)–(11) of [4] inserted a spurious e^{2ky} and solved Eq. (10) incorrectly (under Neumann boundary conditions, the true zero mode is flat $f_0 = C_0$, not double-exponential).

Replacing the invalid 5D APS expression with the **Dai–Freed / Witten** anomaly inflow phase $\exp(-i\pi\eta(D_{Y_5}))$ on an odd 5D bulk Y_5 (whose variation over a 6D mapping torus obeys the 6D APS index theorem), introducing a gauge-invariant 2-form axion-torsion polarization θF so $d(\star F - \theta F) = J_{\rm matter}$, and localizing 5D gauge fields cleanly via a Batell–Gherghetta bulk dilaton kinetic function $Z(y) = e^{-\alpha k y}$ yields Eqs. (1)–(2). Crucially, in 4D $\frac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}d^4x = F\wedge F$, which restores a missing factor of two in the cosmic birefringence angle $\beta = \Delta\theta/\Lambda^{2k}$ (correcting Eq. 2 of [5]). Using proper 95% CL CMB bounds rather than central values ($|\beta| < 0.53^\circ$ from Planck/WMAP, or $|\beta| < 1.03^\circ$ including systematic uncertainties [19]) over $d_{\rm LSS}\approx 14\,\mathrm{Gpc}$, and cleanly separating multipole-dependent isotropic $\beta(\ell)$ (BICEP/Keck XXI [20], $|\Delta\beta(\ell)| < 0.30^\circ$) from anisotropic four-point bounds ($A_{\rm CB} < 0.014\,\mathrm{deg}^2$ BK17; $< 0.010\,\mathrm{deg}^2$ SPT-3G) and COBE/FIRAS (μ, y), Eq. (1) bounds radiative torsional CFJ couplings ($c_{\rm CFJ}\sim\alpha_{\rm EM}/\pi$) to $|S_0| \lesssim 2\times10^{-42}\,\mathrm{GeV}$—eleven orders tighter than torsion-pendulum limits ($|S_\mu| \lesssim 10^{-31}\,\mathrm{GeV}$) and demonstrating why a $4\,\mathrm{km}$ Mach–Zehnder interferometer experiences $1 - V \approx \frac{1}{2}(k_{AF}L)^2 \lesssim 10^{-50}$.

$$S_{\rm eff}=\int_{X_4}\left(-\frac{1}{2}F\wedge\star F+\frac{\theta}{\Lambda^{2k}}F\wedge F\right)\implies\beta=\frac{\Delta\theta}{\Lambda^{2k}},\qquad |k_{AF}|=|c_{\rm CFJ}S_0|\lesssim 4\text{–}8\times10^{-45}\,\text{GeV},\tag{1}$$

$$-\partial_y\big(e^{-2ky}\partial_y f_n(y)\big)=m_n^2 f_n(y),\qquad Z(y)=e^{-\alpha ky}\implies f_0(y)=C_0,\quad \rho_{\rm EM}(y)\propto e^{-\alpha ky}C_0^2,\tag{2}$$

(2) Black hole soft hair, holographic saddle allowability, and quantum extremal surfaces. In [9], covariant phase-space derivations omitted the Einstein–Maxwell stress-energy cancellation $G_{\mu\nu}=8\pi GT^{\rm EM}_{\mu\nu}$ in Eq. (2.8), omitted commutator terms in Eqs. (3.3)–(3.4) so that $K_{\rm EM}$ vanished identically on shell, used $\frac{1}{2}\delta c/c$ instead of the canonical Cardy scaling $\delta S/S=\delta c/c$, miscounted higher-dimensional modes at $r_H\omega_{\rm cut}\sim1$ as $(r_H\omega_{\rm cut})^{D-1}$ instead of $(r_H\omega_{\rm cut})^{D-2}$, and overlooked **Haco, Perry, and Strominger (2019)** [21], who already proved that Kerr–Newman electromagnetic soft hair reproduces *exact* $S_{\rm BH}=A_H/(4G)$ with zero excess entropy (also correcting Ref. [6] of [9] to Hayward, `gr-qc/0004051`, and Ref. [9] to Harris and Kanti, `hep-ph/0309054`). Genuine photon-sector deviations from Bekenstein–Hawking area enter at one loop via **Sen's logarithmic corrections** $\Delta S=c_{\log}(D,N_\gamma)\ln(A_H/G_D)$ and D-dimensional greybody emission spectra.

In holographic cosmology [6–8, 10], [6] cleanly established $dJ_s=0$ and exclusion of local Proca mass on closed-fiber Kaluza–Klein bundles (distinct from Hawking–Hertog dS/CFT; correcting Ref. [5] of [2, 3, 6] to Witten, `arXiv:0905.4795`). Meanwhile, [7] confused AdS Breitenlohner–Freedman stability with dS principal series, dropped $1/L^2$ from the Klebanov–Witten alternate-quantization window $m^2_{\rm BF}\le m^2<m^2_{\rm BF}+L^{-2}$, wrote a divergent $k=0$ spectral-action moment (true value $f_0=f(0)$), and conflated band-structure $m^*<0$ with Higgs $\mu^2<0$. Complex gravitational saddles in no-boundary cosmology should be evaluated via the **Kontsevich–Segal–Witten** allowability criterion $\sum_i|\arg\lambda_i|<\pi$ [22], while [8, 10] are replaced by standard Quantum Extremal Surface (QES) Page-curve error correction (eliminating the conflated $\mathcal{O}(G^0)$ FLM terms, unbounded $-\Lambda_Y(\Psi^\dagger\Psi)^2$ potentials, and unnormalized $|c_n|=1/N$ Wheeler–DeWitt superpositions).

(3) Corrected Kaluza–Klein EYMH–radion equations and a no-go theorem for electric Born–Infeld solitons. Across the four radion-soliton papers [11–14], the ghost-free canonical radion $\phi=M_{\rm Pl}\sqrt{\frac{n(n+2)}{2}}\sigma$ and exponential gauge coupling $f(\phi)=e^{+n\sigma}=e^{-c_1\phi/M_{\rm Pl}}$ ($c_1=-\sqrt{2n/(n+2)}$) in [12] are sound, but five structural bugs broke the coupled ODEs: (i) [11] omitted a bulk cosmological constant, locking its vacuum in AdS_4 ($M_{\rm ADM}=-\infty$), whereas [12] added a spurious constant $+\Lambda_0$ instead of the dimensionally consistent bulk term $+Ce^{-n\sigma}$; (ii) [12] miscalculated the Higgs Weyl exponent $-2n+(n-2)+n=-2$ (giving $y(\phi)=e^{-2\sigma}$ for internal gauge Higgs or $y(\phi)\equiv1$ for a bulk fundamental scalar) and dropped $e^{-n\sigma}$ from $U_{\rm bulk}(\Phi)$; (iii) [11–13] retained a spurious r^2 inside $(r^2NSf(\phi)K')'$, omitted $1/N$ from S'/S, flipped the radion source sign in Eq. (25) of [12], dropped the $(K^2-1)^2$ quartic term from the Derrick virial identity, and wrote a 5×5 instead of 3×3 Regge–Wheeler perturbation system; and (iv) [14] flipped c_1 and wrongly assumed Born–Infeld electrodynamics is conformally invariant (in fact $\beta_{\rm eff}(\sigma)=\beta e^{-n\sigma}$ and $T^t_t\equiv T^r_r\implies S'/S=4\pi x\varphi'^2$). The corrected vacuum and EYMH–radion equations read:

$$V_{\rm eff}(\sigma)=-A\,e^{-n\sigma}+B\,e^{-2n\sigma}+C\,e^{-n\sigma}\quad\text{or}\quad -Ax^{n+2}+Bx^{n+4}+Cx^n,\quad C=\left(1-\frac{1}{n}\right)Ax_0^2\ \ (n\ge2),\tag{3}$$

$$\big(NSf(\phi)K'\big)'=SK\left(e^2y(\phi)H^2+f(\phi)\frac{K^2-1}{r^2}\right),\qquad \frac{S'}{S}=\frac{4\pi Gr}{N}(\rho+p_r),\tag{4}$$

Theorem 1 (No-Go Obstruction for Static Einstein–Radion–Born–Infeld Electric Solitons). *Globally regular, asymptotically flat static spherically symmetric solitons do not exist in the Einstein–radion–Born–Infeld system of [14] for any electric charge $Q\in\mathbb{R}$.*

Proof. For $Q=0$ ($E\equiv0$), $V_{\rm eff}(\phi)\ge0$ implies $\rho+p_r+2p_\theta=-2V_{\rm eff}\le0$, so the Komar identity yields $M_{\rm ADM}=-2\int_{\mathbb{R}^3}e^{\Phi}V_{\rm eff}\,dV\le0$; by the Schoen–Yau Positive Mass Theorem ($M_{\rm ADM}\ge0$), only trivial Minkowski space exists. For $Q\neq0$, Gauss's law $r^2f(\phi)D(E/S)=Q$ forces $E_{\rm phys}\to\beta$ and $D\to\infty$ as $r\to0$, yielding Eq. (5): a conical solid-angle deficit $N(0)=1-8\pi G\beta Q\neq1$ with $R\sim r^{-2}$ curvature singularity and a distributional point charge $4\pi Q\delta^{(3)}(\vec{r})$ at the origin. □

$$\rho_{\rm BI}(r)\sim\frac{\beta Q}{r^2}\ \ (r\to0)\implies m(r)\sim4\pi\beta Q\,r\implies N(0)=1-8\pi G\beta Q\neq1,\tag{5}$$

Regular non-Abelian EYMH–radion solitons governed by Eqs. (3)–(4) remain viable provided $m_\phi\gtrsim4\,\text{meV}$ ($R_0\lesssim50\,\mu\text{m}$) to satisfy Eöt-Wash torsion-balance fifth-force bounds; in the Abelian sector, regularity requires purely *magnetic* Bardeen/Hayward-type NLED cores giving a de Sitter center $N(r)=1-\Lambda_{\rm core}r^2/3$.

(4) No-bounce theorem for Starobinsky gravity and obstructions to $(3+3)$ ultrahyperbolic spacetime. In [16], a nonsingular bounce $a(t)=a_0\sqrt{1+(t/t_0)^2}$ was proposed inside a ghost-free Starobinsky action $S=\int d^4x\sqrt{-g}[\frac{M_{\rm Pl}^2}{2}R+\alpha R^2+\beta R_{\mu\nu}R^{\mu\nu}]+S_m$ (note also $[\alpha']=[\beta']=M^{-2}$ and $M^2_{\rm scalaron}=M^2_{\rm Pl}/(12\alpha)$). In flat FLRW ($C_{\mu\nu\rho\sigma}\equiv0$, $\alpha_{\rm eff}=\alpha+\frac{1}{3}\beta$), exact variation yields the 00-Friedmann equation (6):

$$3M_{\rm Pl}^2H^2+36\alpha_{\rm eff}(2H\ddot{H}-\dot{H}^2+6H^2\dot{H})=\rho_m \implies \rho_m(0)=-36\alpha_{\rm eff}\,\dot{H}(0)^2<0 \quad (\alpha_{\rm eff}>0). \tag{6}$$

At *any* bounce ($H(0)=0, \dot{H}(0)>0$), Eq. (6) forces $\rho_m(0)=-36\alpha_{\rm eff}\dot{H}(0)^2<0$ whenever $\alpha_{\rm eff}>0$, violating the Weak Energy Condition (while $\alpha_{\rm eff}<0$ introduces a tachyonic scalaron or spin-2 ghost). A healthy, BKL-shear-safe ($\rho_{\rm shear}\propto a^{-6}$) nonsingular bounce therefore requires either ekpyrotic contraction ($w\gg 1$) with beyond-Horndeski/DHOST braiding or closed $k=+1$ spatial curvature constrained by $|\Omega_k|<0.005$ (fixing also the wrong $+p_a^2/(2a)$ Wheeler–DeWitt kinetic sign, Huber vs. Mirzakhani geodesic counting, and critical Φ_4^4 triviality in [17]). Similarly, $(3+3)$ ultrahyperbolic geometry [15] is obstructed by: (i) Asgeirsson–John–Craig–Weinstein Cauchy ill-posedness (exponential Hadamard blow-up $e^{\kappa t_1}$); (ii) connected timelike S^2 directions destroying causal ordering; (iii) two negative-norm vector ghosts (absent in Bars' $Sp(2,\mathbb{R})$-gauged 2T-physics); (iv) continuous negative-semidefinite spectrum of Δ_T on $\mathbb{R}^3_T$ failing fermion mass ratios ($1:4.5:21$ vs. observed $1:206.8:3477$); and (v) GW170817 ($|\Delta v_{\rm GW}/c|<10^{-15}$) plus $w(z)=-1+0.05(1+z)^3$ exploding at $z=1100$.

(5) Quantitative cosmological and gravitational constraints. Table 1 summarizes the governing experimental and mathematical bounds across [1–17].

Table 1. Quantitative observational and mathematical constraints across torsional inflow, black hole entropy, Kaluza–Klein solitons, and cosmological bounces.

Sector / Paper Group	Primary Constraint / Theorem	Quantitative Bound	Viable Completion
Torsional CFJ & RS [1–5]	Planck/WMAP + BK XXI + BK17/SPT-3G	$\lvert S_0\rvert \lesssim 2\times10^{-42}$ GeV; $\lvert\Delta\beta(\ell)\rvert<0.30^\circ$	Dai–Freed inflow + dilaton $Z(y)=e^{-\alpha k y}$
Soft Hair & Saddles [7–10]	Haco–Perry–Strominger + KSW allowability	$S_{\rm cl}=A_H/(4G)$; $\sum\lvert\arg\lambda_i\rvert<\pi$	Sen 1-loop $c_{\log}\ln(A_H/G)$ + QES Page curve
KK Solitons [11–14]	Theorem 1 + Eöt-Wash fifth-force limit	$N(0)=1-8\pi G\beta Q$; $m_\phi\gtrsim 4\,$meV	EYMH Eqs. (3–4) or magnetic Bardeen NLED
Bounces & $(3,3)$ [15–17]	Eq. (6) No-Bounce + GW170817 + Planck	$\lvert\Omega_k\rvert<0.005$; $\lvert\Delta v_{\rm GW}/c\rvert<10^{-15}$	Ekpyrotic DHOST or closed $k=+1$ bounce

(6) Request to CMB Cosmology, Relativistic-Gravity, and Holography Groups, and Conclusion. We request that CMB polarization teams refine joint multipole-dependent $\beta(\ell)$ and anisotropic $A_{\rm CB}$ bounds on axion–torsion models (Eq. 1), and that numerical-relativity groups integrate the corrected five-ODE EYMH–radion system (Eqs. 3–4) and its 3×3 Regge–Wheeler stability matrix up to the Breitenlohner–Forgács–Maison collapse limit. Eliminating the form-degree, Weyl-weight, entropy, and flat-bounce obstructions above places all viable sectors of [1–17] on firm mathematical and observational ground.

References

[1] P. De Ceuster, *Investigating an Unobserved Photonic Law (Paper I of III): Holographic Boundary States from Optics to String Theory*, Zenodo (2026), https://doi.org/10.5281/zenodo.21557661.

[2] P. De Ceuster, *Investigating an Unobserved Photonic Law (Paper II of III): Holographic Photon Continuity: A Topological Extension of Maxwell's Equations*, Zenodo (2026), https://doi.org/10.5281/zenodo.21557719.

[3] P. De Ceuster, *Investigating an Unobserved Photonic Law (Paper III of III): Torsional Anomaly Inflow: Riemann-Cartan Holography and the Protected Photon*, Zenodo (2026), https://doi.org/10.5281/zenodo.21557769.

[4] P. De Ceuster, *Investigations into an unobserved photonic law of nature: Torsional Boundary States in Warped Compactifications: An Effective APS-Type Index Formula with Nieh-Yan Contributions (Paper IV)*, Zenodo (2026), https://doi.org/10.5281/zenodo.21598615.

[5] P. De Ceuster, *Investigations into an unobserved photonic law of nature: Quantitative Observational Bounds from the CMB: Constraining Torsional Anomaly Inflow (Paper V)*, Zenodo (2026), https://doi.org/10.5281/zenodo.21599311.

[6] P. De Ceuster, *A Fiber-Integration Extension of Maxwell's Equations on the Boundary of a Holographic Bulk inspired by Hertog*, Zenodo (2025), https://doi.org/10.5281/zenodo.17850108.

[7] P. De Ceuster, *Critical Divergence in Holographic Foundations: Conflict with the French School, Ill-posed Relation with Negative Effective Mass, and the Higgs*, Zenodo (2026), https://doi.org/10.5281/zenodo.18232578.

[8] P. De Ceuster, *A new photonic law of nature: the Great Decoherence Paradox. On Holographic Entanglement Horizons and Trans-Dimensional Photonic Continuity*, Zenodo (2026), https://doi.org/10.5281/zenodo.18944763.

[9] P. De Ceuster, *Identifying Theoretical Gaps in Hawking's Black Hole Radiation Model in Light of Modern Photonics and Higher-Dimensional Photon Propagation : Photon-Coupled Soft Hair and Higher-Dimensional Photon Channels A Critical Extension of the Soft-Hair Formalism*, Zenodo (2025), https://doi.org/10.5281/zenodo.16783235.

[10] P. De Ceuster, *The Holographic State Space: A Mathematical Formalism for the Cosmic Storehouse: Proto-theory*, Zenodo (2025), https://doi.org/10.5281/zenodo.16768371.

[11] P. De Ceuster, *Radion-Supported Gravitating Solitons in Flux-Stabilized Kaluza–Klein Compactifications*, Zenodo (2026), https://doi.org/10.5281/zenodo.21309825.

[12] P. De Ceuster, *A Theoretical Framework for Radion-Supported Gravitating Solitons: Towards a Geometric Interpretation of Confined Gauge-Field Mass*, Zenodo (2026), https://doi.org/10.5281/zenodo.21313016.

[13] P. De Ceuster, *A Framework for Radion-Supported Gravitating Solitons: Toward a Geometric Interpretation of Localized Gravitational Mass*, Zenodo (2026), https://doi.org/10.5281/zenodo.21319638.

[14] P. De Ceuster, *Toward Geometric Localization in Kaluza-Klein Gravity: Einstein-Radion Nonlinear Electromagnetic Solitons*, Zenodo (2026), https://doi.org/10.5281/zenodo.21324365.

[15] P. De Ceuster, *On the Geometric and Spectral Inevitability of Ultrahyperbolic (3 + 3) Spacetime: Mathematical Foundations, Temporal Eigenvalue Spectra, and Unified Gauge Phenomenology*, Zenodo (2026), https://doi.org/10.5281/zenodo.21939787.

[16] P. De Ceuster, *G-Theory → Maxwell Duality: Lie-N Compactification, a Nonsingular Bounce, and Topological Photon Signatures : Prospects for evidence of an unobserved extended Maxwell law*, Zenodo (2025), https://doi.org/10.5281/zenodo.17217166.

[17] P. De Ceuster, *Dreams of the multiverse- Topological Recursion in Nested Cyclic Multiverses, Three actors: The "Basket-Balloon" Metric, The Hiranyagarbha Seed, and Standard Model Renormalization*, Zenodo (2026), https://doi.org/10.5281/zenodo.18196472.

[18] O. Chandía and J. Zanelli, Phys. Rev. D 55, 7580 (1997); J. Nissinen and G. E. Volovik, Phys. Rev. Res. 2, 033269 (2020).

[19] P. Diego-Palazuelos *et al.*, Phys. Rev. Lett. 128, 091302 (2022); R. S. Sullivan *et al.*, arXiv:2502.07654 (2025).

[20] P. A. R. Ade *et al.* (BICEP/Keck Collaboration XXI), arXiv:2603.06812 (2026); BICEP/Keck XVII, Phys. Rev. Lett. 130, 121001 (2023).

[21] S. Haco, M. J. Perry, and A. Strominger, *Kerr-Newman Black Hole Entropy and Soft Hair*, JHEP 05, 056 (2019), arXiv:1902.02247.

[22] M. Kontsevich and G. Segal, Quart. J. Math. 72, 673–699 (2021); E. Witten, arXiv:2111.06514 (2021).