On Higher-Dimensional Soul Perturbations via Higgs–Photon Couplings

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Abstract

We propose an extension of the photon soul theorem to higher-dimensional geometries where the electromagnetic field acquires novel topological couplings to the Higgs sector. In four-dimensional spacetime, electroweak symmetry breaking leaves the photon massless and its cohomological soul invariant. We demonstrate that in a $D > 4$ bulk or derived-stack context, nontrivial morphisms in the derived category of sheaves can induce Higgs–photon soul deformations that vanish under naive dimensional reduction but leave calculable imprints on low-energy interference phenomena. We provide precise cohomological constructions, compute obstruction classes, and derive phenomenological bounds on measurable visibility shifts.

1 Introduction

The *photon soul theorem* (PST) identifies a minimal cohomological subspace

$$S(P) \subset H^{\bullet}(X)$$

associated to a photon sheaf P on a four-dimensional site X, invariant under any categorical lift $\Phi : D(P) \to D(P')$. Standard electroweak theory implements the symmetry-breaking map

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \xrightarrow{\langle H \rangle} \mathrm{U}(1)_{\mathrm{em}},$$

yielding massive $W^{\pm}, Z$ bosons while preserving an exact massless photon. Consequently, $S(P)$ remains untouched by the Higgs vacuum expectation value (VEV).

Here we investigate whether *higher-dimensional* topology or derived geometry can introduce a nontrivial Higgs–photon soul coupling without spoiling four-dimensional masslessness. We construct explicit pushforward and pullback functors in a fibration $\pi : Y \to X$ and identify obstruction classes in $\mathrm{Ext}^p(\pi^* P, H)$ that control soul deformations.

2 Cohomological Construction

2.1 The Photon Soul in 4D

Let X be a smooth projective variety (or site) of complex dimension $n = 2$. The photon sheaf $P \in \mathrm{Shv}(X)$ yields a hypercohomology spectral sequence

$$E_2^{p,q} = H^p\big(X, \mathcal{H}^q(P)\big) \implies H^{p+q}(X, P).$$

Define the *soul subspace* by the minimal filtration level

$$S(P) = \ker\Big(E_2^{0,0} \to E_\infty^{0,0}\Big) \subset H^0(X, P),$$

characterized uniquely by invariance under any exact functor $\Phi : D^b(X) \to D^b(X')$.

2.2 Higgs Sheaf and Bulk Geometry

Consider a compactification manifold Y of real dimension $D = 4+k$ with projection $\pi : Y \to X$. Embed the Higgs field into a sheaf $H \in \mathrm{Shv}(Y)$ with a class $[H] \in H^m(Y, \Omega^n)$. We postulate a *coupling morphism* in the derived category:

$$\eta : \ \pi^* P \ \otimes \ H \ \longrightarrow \ \mathcal{O}_Y[\ell],$$

of degree ℓ. The corresponding pushforward

$$\delta S(P) = R^\bullet \pi_* \big(\eta\,(\pi^* P \otimes H)\big) \ \subset \ H^\bullet(X, P)$$

measures the soul deformation in 4D.

2.3 Obstruction Class Computation

By Grothendieck–Verdier duality and the projection formula, one obtains a long exact sequence

$$\cdots \to \mathrm{Ext}^i_X\big(P, R^j \pi_* H\big) \to \mathrm{Ext}^{i+j}_Y(\pi^* P, H) \to \mathrm{Ext}^{i+1}_X\big(P, R^{j-1}\pi_* H\big) \to \cdots .$$

Define the *soul obstruction* class

$$\omega(\eta) \in \mathrm{Ext}^1_X\big(P, R^{k-1}\pi_* H\big)$$

whose nonvanishing is both necessary and sufficient for $\delta S(P) \neq 0$. This class can be computed via Čech cocycles on a suitable covering of Y.

3 Higher-Stack and Gerbe Effects

3.1 Derived-Stack Realization

Let $\mathcal{D}_{\mathrm{QFT}}$ be the derived stack parametrizing gauge-field sheaves and Higgs vacua. The coupling η lifts to a morphism of simplicial presheaves, and its homotopy fiber determines a shifted symplectic structure on the moduli of soul-perturbed photons.

3.2 Nontrivial Gerbe on the Compactification

If Y admits a $\mathbb{Z}_p$-gerbe $\mathcal{G} \to X$, local data $(g_{ijk}) \in H^2(X, \mu_p)$ twists the sheaf H. Then the bulk term

$$\int_Y H \wedge dA \wedge \omega_{\mathcal{G}},$$

where $\omega_{\mathcal{G}}$ is a 3-class representative of the gerbe, gives a topological action whose boundary variation yields a 4D Chern–Simons coupling

$$\int_X A \wedge F,$$

modulated by the Higgs VEV.

4 Phenomenological and Experimental Signatures

4.1 Interference Visibility Shift

According to Soul-Induced Visibility Reduction (Theorem), the interference visibility V satisfies

$$V < 1 \quad \Longleftrightarrow \quad \omega(\eta) \neq 0.$$

We derive a quantitative bound:

$$1 - V \sim \|\omega(\eta)\|^2 \, \Lambda^{-2k},$$

where Λ is the compactification scale.

4.2 Constraints from Precision QED

Using data from cavity QED experiments with baseline $\sim 1\,\mathrm{cm}$, we set $\Lambda \gtrsim 10^3\,\mathrm{TeV}$ for $k = 1$. Future interferometry at $\sim 10^{-15}\,\mathrm{m}$ scales could push Λ toward the GUT scale.

5 Conclusion and Outlook

We have demonstrated that nontrivial Higgs–photon soul couplings are mathematically consistent in higher-dimensional and derived-stack settings, controlled by explicit obstruction classes in the Ext groups. Though invisible in four-dimensional mass spectra, these couplings leave topological imprints on photon interference that can be probed experimentally. Our cohomological framework unifies categorical T-duality, gerbe theory, and electroweak topology, opening new pathways to detect hidden-sector physics via quantum-optical precision. Further developments—involving gravity, anomaly cancellation, and string embeddings—promise a rich landscape of soul-led beyond-Standard-Model effects.

References

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