Topological Electroweak Unification: Higher-Dimensional Photonic Sheaves, Chiral Index Invariants, and the Extended Standard Model

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Abstract

Rejecting the Standard Model (SM) is easily done as we have showed -to great personal satisfaction -in our previous works; however, a constructive theoretical paradigm must fundamentally improve, complete, and geometrically ground it [1]. In this work, we formulate an extended geometric framework for the Standard Model by asserting that physical four-dimensional spacetime X represents the base manifold of a smooth, oriented fiber bundle $\pi : Y \to X$ with a compact, boundaryless k-dimensional Riemannian spin manifold K as fiber ($\dim_{\mathbb{R}} Y = 4 + k, k \in 2\mathbb{N}$) [1]. We model the electroweak gauge sector as a holomorphic principal $G^{\mathbb{C}}_{EW}$-bundle ($G^{\mathbb{C}}_{EW} = SL(2, \mathbb{C}) \times \mathbb{C}^*$) with associated adjoint bundle $\mathrm{ad}(P_{EW})$ and coherent adjoint sheaf $\mathcal{E}_{EW} = \mathcal{O}_Y(\mathrm{ad}(P_{EW})) \in \mathrm{Coh}(Y)$ [1]. Following spontaneous symmetry breaking $\langle \mathcal{H} \rangle = v_B \neq 0$, the unbroken photonic reality is formalized as the holomorphic line subbundle $\mathcal{L}^{\mathrm{ad}}_{\gamma} \subset \mathrm{ad}(P_{EW})$ stabilized by the electric charge generator $Q = T^3 + Y_W$, with associated coherent sheaf $\mathcal{E}_{\gamma} = \mathcal{O}_Y(\mathcal{L}^{\mathrm{ad}}_{\gamma})$ [1].

To ensure dimensional consistency on the smooth $(4 + k)$-manifold Y, the bulk photon-Higgs topological coupling is formulated in differential cohomology as an integral character $\widehat{\eta} \in \widehat{H}^{k+3}(Y)$ with closed curvature representative $\mathcal{J} = \mathrm{curv}(\widehat{\eta}) \in \Omega^{k+3}_{\mathrm{cl},\mathbb{Z}}(Y)$ satisfying $d_Y \mathcal{J} = 0$, constructed from the invariant characteristic forms of the gauge and Higgs connections [1, 2]. Fiber integration along K induces a closed conserved 3-form soul current on spacetime:

$$J_s := \pi_* \mathcal{J} = \int_K \mathcal{J} \in \Omega^3(X), \qquad d_X J_s = \pi_*(d_Y \mathcal{J}) = 0. \tag{1}$$

The degree-consistent extended Maxwell equations are formulated as $dF = 0$ and $d * F = J_s + eJ_{\mathrm{em}}$, yielding a localized Gauss-law topological charge $Q_{\mathrm{soul}}(\Sigma^3) = \int_{\Sigma^3} J_s = \int_{\partial\Sigma^3} *F - e \int_{\Sigma^3} J_{\mathrm{em}}$ for any spatial 3-chain $\Sigma^3 \subset X$ with boundary $\partial\Sigma^3$ [1, 4]. Under the boundary condition $\mathcal{J}_{k=0} \equiv 0$, the framework identically recovers standard four-dimensional electrodynamics [1].

Beyond abelian electrodynamics, we demonstrate that: (1) on a compact spin manifold K, the internal Dirac operator $\mathcal{D}_K$ yields a net chiral generation index $\mathrm{ind}(\mathcal{D}_K) = \int_K \widehat{A}(TK) \wedge \mathrm{ch}(\mathcal{V}_F) = 3$ when $c_1(\mathcal{V}_F) = 0$ and $\frac{1}{2} \int_K c_3(\mathcal{V}_F) = 3$, providing a topological realization of a net chiral index of three; (2) coupling J_s into the electroweak neutral sector induces scale-dependent corrections to the effective weak mixing angle $\sin^2 \theta^{\mathrm{eff}}_W(q^2)$; (3) compactification scale boundary conditions Λ serve as an effective field theory threshold regularizing the Higgs mass against unconstrained quadratic ultraviolet divergences; and (4) topological holonomies across spatial 3-chains predict a leading-order visibility suppression in precision quantum interferometry $1 - V \approx \frac{1}{2}\langle(\Delta\phi_s)^2\rangle \sim \|\omega(\eta)\|^2 \Lambda^{-2k}$ [1]. We establish the differential-cohomological lifting conditions in Cheeger-Simons cohomology [2], formulate bulk Green-Schwarz anomaly-consistency requirements, and outline experimental tests spanning precision optical interferometry, $(g - 2)_\mu$, and atomic parity violation.

1 Introduction

The Standard Model of particle physics constitutes one of the most successful empirical frameworks in modern science, providing exquisite predictions across electroweak gauge interactions, precision quantum electrodynamics, and Higgs condensation [4]. Despite these successes, fundamental questions remain unresolved: the origin of the three chiral fermion families, the hierarchy problem and the radiative stability of the electroweak scale, the geometric nature of electroweak symmetry breaking, and the topological quantization of abelian gauge charge [1, 3].

Rather than discarding the Standard Model, constructive progress demands generalizing its underlying geometric and algebraic substrate [1]. In this treatise, we propose that four-dimensional spacetime X represents the base manifold of a $(4+k)$-dimensional bulk space Y fibered by a compact internal Riemannian spin manifold K [1]. Within this higher-dimensional setting, physical gauge fields and Higgs interactions are formalized through coherent sheaves and differential characters on Y, whose pushforward under fiber integration yields low-energy field theories on X [1, 2].

This work develops an integrated electroweak unification program:

1. **Adjoint Electroweak Sheaves**: We formulate the $SU(2)_L \times U(1)_Y$ gauge sector within the holomorphic adjoint bundle $\mathrm{ad}(P_{EW})$, where physical electrodynamics emerges as the unbroken line subbundle $\mathcal{L}_\gamma^{\mathrm{ad}}$ stabilized by $Q = T^3 + Y_W$ following bulk Higgs condensation [1].

2. **Topological Origin of Three Generations**: We express the net chiral generation count as the topological index $\mathrm{ind}(\mathcal{D}_K) = 3$ of the internal spin Dirac operator with $c_1(\mathcal{V}_F) = 0$.

3. **Modified Electroweak Neutral Currents**: We couple the closed topological 3-form current $J_s = \pi_* \mathcal{J}$ to the physical photon and Z^0 fields, deriving scale-dependent shifts in the Weinberg angle $\sin^2 \theta_W$ and the muon anomalous magnetic moment [1].

4. **EFT Threshold Regularization of the Hierarchy**: We show that internal Kaluza-Klein geometry provides a physical threshold Λ regularizing radiative Higgs mass corrections.

5. **Quantum Optical Interferometry Signatures**: We evaluate topological phase fluctuations on spatial chains, deriving the testable visibility suppression $1 - V \sim \|\omega(\eta)\|^2 \Lambda^{-2k}$ [1].

2 Geometric and Sheaf-Theoretic Framework

Let X denote a 4-dimensional, smooth, oriented, time-orientable pseudo-Riemannian spacetime manifold. We define the bulk space Y as the total space of a smooth fiber bundle:

$$K \hookrightarrow Y \xrightarrow{\pi} X \tag{2}$$

where the fiber K is a compact, connected, boundaryless k-dimensional Riemannian spin manifold $(w_2(TK) = 0, \partial K = \emptyset)$, fixing $\dim_{\mathbb{R}} Y = 4 + k$ [1]. To ensure categorical compatibility with coherent sheaf theory on the complexified sectors, we endow Y with the structure of a complex analytic manifold with even fiber dimension $k \in 2\mathbb{N}$.

2.1 Holomorphic Principal Bundle and Adjoint Construction

Let $P_{EW} \to Y$ be a holomorphic principal $G_{EW}^{\mathbb{C}}$-bundle over Y, where $G_{EW}^{\mathbb{C}} = SL(2, \mathbb{C}) \times \mathbb{C}^*$ represents the complexified Standard Model electroweak gauge group. The bundle P_{EW} admits a smooth reduction of structure group to its maximal compact subgroup $G_{EW} = SU(2)_L \times U(1)_Y$.

Definition 1 (Electroweak Adjoint Sheaf and Curvature)**.** *Let* $\mathrm{ad}(P_{EW}) = P_{EW} \times_{\mathrm{Ad}} \mathfrak{g}_{EW}^{\mathbb{C}}$ *be the holomorphic adjoint vector bundle on* Y*, where* $\mathfrak{g}_{EW}^{\mathbb{C}} = \mathfrak{sl}(2, \mathbb{C}) \oplus \mathbb{C}$*. We define:*

1. *The electroweak adjoint coherent sheaf* $\mathcal{E}_{EW} := \mathcal{O}_Y(\mathrm{ad}(P_{EW})) \in \mathrm{Coh}(Y)$ *as the locally free sheaf of holomorphic sections of the adjoint bundle [1].*
2. *The underlying smooth gauge connection 1-form* $\mathcal{A}_{EW} = \mathcal{W}^a T^a + \mathcal{B} Y_W \in \Omega^1(Y, \mathfrak{su}(2) \oplus \mathfrak{u}(1))$*, with globally defined curvature 2-form* $\mathcal{F}_{EW} = d\mathcal{A}_{EW} + \mathcal{A}_{EW} \wedge \mathcal{A}_{EW} \in \Omega^2(Y, \mathfrak{g}_{EW})$ *satisfying the non-abelian Bianchi identity:*
$$D_{EW}\mathcal{F}_{EW} := d\mathcal{F}_{EW} + [\mathcal{A}_{EW}, \mathcal{F}_{EW}] = 0. \tag{3}$$
3. *The bulk Higgs sheaf* $\mathcal{H} = \mathcal{O}_Y(\mathcal{V}_H) \in \mathrm{Coh}(Y)$*, where* $\mathcal{V}_H$ *transforms in the fundamental* $(2, 1/2)$ *representation of* G_{EW} *[1].*

2.2 Electroweak Symmetry Breaking and the Photonic Projection

Spontaneous symmetry breaking occurs when the bulk Higgs section acquires a non-zero vacuum expectation value:

$$\langle \mathcal{H} \rangle = \begin{pmatrix} 0 \\ \frac{v_B}{\sqrt{2}} \end{pmatrix}, \qquad v_B \in \mathbb{R}^+. \tag{4}$$

This vacuum profile breaks the algebra $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y \to \mathfrak{u}(1)_{\mathrm{em}}$ generated by the electric charge operator $Q = T^3 + Y_W$.

Definition 2 (Photonic Projection Sheaf)**.** *The unbroken electromagnetic subbundle is the holomorphic line subbundle* $\mathcal{L}_\gamma^{\mathrm{ad}} \subset \mathrm{ad}(P_{EW})$ *defined as the stabilizer subbundle of* $\langle \mathcal{H} \rangle$*, spanned pointwise by* Q*. We define the coherent photonic projection sheaf:*

$$\mathcal{E}_\gamma = \mathcal{O}_Y(\mathcal{L}_\gamma^{\mathrm{ad}}) \in \mathrm{Coh}(Y)1001[1]. \tag{5}$$

The connection on $\mathcal{L}_\gamma^{\mathrm{ad}}$ *is locally represented by* $\mathcal{A}_\gamma = \sin\theta_W \mathcal{W}^3 + \cos\theta_W \mathcal{B}$*, with abelian field strength* $\mathcal{F}_\gamma = d\mathcal{A}_\gamma \in \Omega^2_{\mathrm{cl}}(Y)$ *[1].*

3 Topological Characteristic Classes and the Spacetime Soul Current

Because $\dim_{\mathbb{R}} Y = 4 + k$, the exterior algebra $\Omega^\bullet(Y)$ naturally supports differential forms of degree up to $4 + k$. Since $k + 3 \leq 4 + k$ holds for all physical dimensions $k \geq 0$, the bulk topological interaction form lives naturally in the $(k+3)$-th de Rham cohomology group of the smooth underlying $(4 + k)$-manifold Y [1].

Geometric Postulate 1 (Bulk Topological Characteristic Form)**.** *Let* $\mathcal{A}_\gamma$ *and* $\mathcal{A}_H$ *denote the connections associated with* $\mathcal{E}_\gamma$ *and the Higgs bundle* $\mathcal{V}_H$*. We postulate that the invariant Chern-Simons and Atiyah-Chern coupling between the unbroken photonic connection and the Higgs geometry determines an integral, closed differential* $(k + 3)$*-form:*

$$\mathcal{J} \in \Omega^{k+3}_{\mathrm{cl},\mathbb{Z}}(Y), \qquad d_Y\mathcal{J} = 01001[1]. \tag{6}$$

The form $\mathcal{J}$ *represents a characteristic topological class* $[\mathcal{J}] \in H^{k+3}_{\mathrm{dR},\mathbb{Z}}(Y)$*.*

Definition 3 (Fiber Pushforward Soul Current)**.** *The physical 3-form soul current* J_s *on spacetime* X *is defined via integration along the fiber* K*:*

$$J_s := \pi_*\mathcal{J} = \int_K \mathcal{J} \in \Omega^3(X)1001[1]. \tag{7}$$

Theorem 1 (Conservation of the Pushforward Current)**.** *Let $\pi : Y \to X$ be a smooth bundle with compact, boundaryless fiber K ($\partial K = \emptyset$), and let $\mathcal{J} \in \Omega^{k+3}_{\mathrm{cl}}(Y)$. Then the spacetime current $J_s \in \Omega^3(X)$ is strictly closed:*

$$d_X J_s = 01001[1]. \tag{8}$$

Proof. Applying the fiberwise Stokes theorem for smooth fiber bundles:

$$d_X J_s = d_X(\pi_* \mathcal{J}) = \pi_*(d_Y \mathcal{J}) + (-1)^{\deg(\mathcal{J})-k} \int_{\partial K} \mathcal{J}. \tag{9}$$

Because $d_Y \mathcal{J} = 0$ by Postulate 1 and $\partial K = \emptyset$, both terms vanish identically:

$$d_X J_s = \pi_*(0) + 0 = 01001[1]. \tag{10}$$

Hence, J_s is a closed differential 3-form on physical spacetime X [1]. □

Proposition 1 (Decoupling in the Four-Dimensional Limit)**.** *When $k = 0$, $Y \cong X$ and $\pi = \mathrm{id}_X$ [1]. Imposing the explicit decoupling condition $\mathcal{J}_{k=0} \equiv 0$ yields $J_s \equiv 0$, identically recovering source-free classical electrodynamics [1].*

4 Topological Chiral Index and Matter Representations

In this framework, chiral fermion families originate from the zero-mode topology of the Dirac operator over the internal spin manifold K.

Definition 4 (Bulk Dirac Operator)**.** *Let K be a compact, oriented Riemannian spin manifold ($w_2(TK) = 0$), and let Ψ be a bulk Dirac spinor on Y taking values in a twisting holomorphic vector bundle $\mathcal{V}_F \to Y$. The $(4 + k)$-dimensional Dirac operator decomposes as:*

$$i\not{D}_Y = i\gamma^\mu D^{(X)}_\mu \otimes \mathbb{I} + \gamma_5 \otimes i\mathcal{D}_K \tag{11}$$

where $\mathcal{D}_K = \sum_{a=1}^k \Gamma^a D^{(K)}_a$ is the internal spin Dirac operator on the fiber K coupled to the background gauge connection and internal spin connection.

Theorem 2 (Chiral Index Invariant on Spin Fibers)**.** *Assuming K is a compact, boundaryless Riemannian spin manifold, the net chiral index $n_L - n_R$ of zero modes of the internal Dirac operator $\mathcal{D}_K$ is given by the Atiyah-Singer index theorem:*

$$\mathrm{ind}(\mathcal{D}_K) = n_L - n_R = \int_K \widehat{A}(TK) \wedge \mathrm{ch}(\mathcal{V}_F) \tag{12}$$

where $\widehat{A}(TK)$ is the Dirac A-roof genus of the tangent bundle TK, and $\mathrm{ch}(\mathcal{V}_F)$ is the Chern character of the twisting flavor vector bundle.

Proof. By the Atiyah-Singer Index Theorem for Dirac operators on compact spin manifolds:

$$\mathrm{ind}(\mathcal{D}_K) = \dim\ker(\mathcal{D}^+_K) - \dim\ker(\mathcal{D}^-_K) = \int_K \widehat{A}(TK) \wedge \mathrm{ch}(\mathcal{V}_F). \tag{13}$$

Each left-handed zero mode $\psi^{(0)}_{L,j}$ ($j = 1, \ldots, n_L$) of $\mathcal{D}_K$ couples to 4D spacetime spinors, generating 4D chiral fermion states prior to electroweak Yukawa condensation. □

Proposition 2 (Topological Realization of Three Generations). *Let K be a 6-dimensional Calabi-Yau manifold ($k = 6, c_1(K) = 0, w_2(TK) = 0$) equipped with a stable holomorphic vector bundle $\mathcal{V}_F$ satisfying $c_1(\mathcal{V}_F) = 0$ and $\frac{1}{2}\int_K c_3(\mathcal{V}_F) = 3$. Then the $c_1(\mathcal{V}_F)c_2(TK)$ contribution vanishes identically, yielding:*

$$\mathrm{ind}(\mathcal{D}_K) = \frac{1}{2}\int_K c_3(\mathcal{V}_F) = 3. \tag{14}$$

This provides a topological realization of a net chiral index of three; identifying this invariant with the three physical Standard Model generations requires an explicit embedding of fundamental fermion representations and the absence or mass-pairing of vector-like zero modes.

5 Modified Electroweak Neutral Currents and Field Equations

Following electroweak breaking, physical spacetime contains the electromagnetic field strength $F \in \Omega^2(X)$ and the massive neutral Z^0 boson 1-form $Z \in \Omega^1(X)$.

Definition 5 (Extended Electroweak Field Equations). *The coupled equations of motion for the physical photon and Z^0 fields in the presence of $J_s \in \Omega^3(X)$ and conventional matter currents are:*

$$dF = 01001[1], \tag{15}$$

$$d * F = J_s + eJ_{\mathrm{em}}1001[1, 4], \tag{16}$$

$$d * Z_2 + M_Z^2 * Z = J_Z + \epsilon_Z J_s \tag{17}$$

where $Z_2 = dZ \in \Omega^2(X)$, $J_{\mathrm{em}} \in \Omega^3(X)$ and $J_Z \in \Omega^3(X)$ are the Standard Model 3-form matter currents, and $\epsilon_Z = \tan\theta_W$ parameterizes the topological neutral-current mixing.

Theorem 3 (Gauss-Law Topological Soul Charge). *Let $\Sigma^3 \subset X$ be a spatial 3-chain with smooth boundary $\partial\Sigma^3$. The total enclosed topological soul charge is:*

$$Q_{\mathrm{soul}}(\Sigma^3) = \int_{\Sigma^3} J_s = \int_{\partial\Sigma^3} *F - e\int_{\Sigma^3} J_{\mathrm{em}}1001[1]. \tag{18}$$

In matter-free regions ($J_{\mathrm{em}} = 0$), $Q_{\mathrm{soul}}(\Sigma^3) = \int_{\partial\Sigma^3} *F$ [1].

Proof. Integrating $d * F = J_s + eJ_{\mathrm{em}}$ over Σ^3 and applying Stokes' theorem on X:

$$\int_{\Sigma^3} J_s = \int_{\Sigma^3}(d * F - eJ_{\mathrm{em}}) = \int_{\partial\Sigma^3} *F - e\int_{\Sigma^3} J_{\mathrm{em}}1001[1]. \tag{19}$$

□

Remark 1. *If Σ^3 is a closed 3-cycle ($\partial\Sigma^3 = \emptyset$), then $\int_{\Sigma^3} J_s = 0$ in the absence of net enclosed matter charge, confirming that $J_s = d * F$ acts as a localized source of differential flux without generating unphysical net global monopoles on compact topologies [1].*

5.1 Corrections to the Effective Weak Mixing Angle

Because the topological current J_s couples simultaneously to the electromagnetic and neutral sectors, the effective low-energy Weinberg angle $\sin^2\theta_W^{\mathrm{eff}}$ receives a scale-dependent correction:

$$\sin^2\theta_W^{\mathrm{eff}}(q^2) = \sin^2\theta_W^{\mathrm{SM}}(q^2) + \Delta_{\mathrm{topo}}(q^2) \tag{20}$$

where

$$\Delta_{\mathrm{topo}}(q^2) = \frac{\epsilon_Z \cos\theta_W \sin\theta_W}{M_Z^2 - q^2}\left(\frac{\|\omega(\eta)\|}{\Lambda^k}\right)^2 1001[1]. \tag{21}$$

This correlates optical interferometric phase shifts directly with precision electroweak neutral-current measurements.

6 Effective Field Theory, Action Formalism, and Hierarchy Constraints

We formulate the 4D low-energy effective field theory on X by coupling the physical potentials $A \in \Omega^1(X)$ and $Z \in \Omega^1(X)$ directly to the 3-form sources J_s, J_{em}, and J_Z.

Definition 6 (Low-Energy Effective Action)**.** *Using differential-form kinetic normalizations ($F \wedge *F = \frac{1}{2}F_{\mu\nu}F^{\mu\nu}\text{vol}_X$), the gauge action is:*

$$S_{\text{eff}}[A, Z] = \int_X \left(-\frac{1}{2}F \wedge *F - \frac{1}{2}Z_2 \wedge *Z_2 - \frac{1}{2}M_Z^2 Z \wedge *Z + A \wedge J_s + \epsilon_Z Z \wedge J_s + eA \wedge J_{\text{em}} + Z \wedge J_Z\right) 1001[1]. \tag{22}$$

Proposition 3 (Exact $U(1)_{\text{em}}$ Gauge Invariance)**.** *$S_{\text{eff}}[A, Z]$ is strictly invariant under $A \mapsto A + d\lambda$ for any gauge parameter $\lambda \in C_c^\infty(X)$ assuming matter current conservation ($d_X J_{\text{em}} = 0$).*

Proof. Under $A \mapsto A + d\lambda$, the variation of the electromagnetic interaction terms is:

$$\delta_\lambda S_{\text{eff}} = \int_X d\lambda \wedge (J_s + eJ_{\text{em}}) = \int_X d\big(\lambda(J_s + eJ_{\text{em}})\big) - \int_X \lambda\, d(J_s + eJ_{\text{em}}). \tag{23}$$

By Stokes' theorem, the boundary term vanishes for compactly supported λ. Because $d_X J_s = 0$ by Theorem 1 and $d_X J_{\text{em}} = 0$ by classical matter conservation, the bulk integral vanishes identically:

$$\delta_\lambda S_{\text{eff}} = 0. \tag{24}$$

Hence, the topological current coupling preserves exact $U(1)_{\text{em}}$ gauge invariance on X [1]. □

6.1 Kaluza-Klein Geometry as an Effective Threshold Regularizer

In the $(4 + k)$-dimensional bundle Y, the compact Riemannian fiber K discretizes internal momentum modes into a discrete Kaluza-Klein spectrum $m_{\vec{n}}^2 = \lambda_{\vec{n}}/R^2 \sim \lambda_{\vec{n}}\Lambda^2$.

EFT Hypothesis 1 (EFT Threshold Regularization Hypothesis)**.** *We assume that the low-energy effective field theory is valid up to the physical compactification scale Λ, where bulk degrees of freedom become dynamical. Under this threshold condition, the one-loop radiative correction to the Higgs mass is regularized by Λ:*

$$\delta m_H^2 \approx \mathcal{O}(1)\frac{g^2}{16\pi^2}\Lambda^2 \ln\left(\frac{\Lambda^2}{m_H^2}\right). \tag{25}$$

The physical geometric scale $\Lambda \sim \mathcal{O}(1\text{–}10\,\text{TeV})$ acts as an effective cutoff parameter, stabilizing the electroweak scale against unconstrained Planck-scale divergences.

7 Differential Cohomology and Anomaly Consistency

To ensure the consistent quantization of topological flux, the gauge connections and the bulk current $\mathcal{J}$ are embedded into Cheeger-Simons differential cohomology $\widehat{H}^\bullet(Y)$ [2].

Geometric Postulate 2 (Differential Cohomological Lift)**.** *We assume that the closed form $\mathcal{J}$ has integral periods ($\mathcal{J} \in \Omega_{\mathbb{Z}}^{k+3}(Y)$) and therefore admits a smooth differential character lift $\widehat{\eta} \in \widehat{H}^{k+3}(Y)$ with curvature:*

$$\text{curv}(\widehat{\eta}) = \mathcal{J}1001[1]. \tag{26}$$

The fiber integration pushforward in differential cohomology $\pi_* : \widehat{H}^{k+3}(Y) \to \widehat{H}^3(X)$ commutes with the curvature operator [2]:

$$\mathrm{curv}(\pi_*\widehat{\eta}) = \pi_*(\mathrm{curv}(\widehat{\eta})) = \pi_*\mathcal{J} = J_s \in \Omega^3_{\mathrm{cl}}(X)1001[1]. \tag{27}$$

Consequently, the differential character $\pi_*\widehat{\eta} \in \widehat{H}^3(X)$ evaluates to quantized holonomies on integral 2-cycles, with curvature J_s, whose integral over spatial 3-chains determines the corresponding boundary holonomy on $\partial\Sigma^3$ [2].

7.1 Bulk Green-Schwarz Anomaly Consistency

For a complete $(4+k)$-dimensional bulk field theory with chiral fermions, gauge and gravitational anomalies are described by the anomaly $(k+6)$-form polynomial $I_{k+6}(\mathcal{F}_{EW}, \mathcal{R})$. Quantum consistency requires the anomaly polynomial to satisfy a generalized Green-Schwarz factorization:

$$I_{k+6}(\mathcal{F}_{EW}, \mathcal{R}) = \sum_a X_a^{(p_a)} \wedge Y_a^{(k+6-p_a)} \tag{28}$$

where X_a and Y_a are invariant polynomials in the gauge curvature $\mathcal{F}_{EW}$ and Riemann curvature $\mathcal{R}$. Any ultraviolet completion of this framework must specify a bulk chiral fermion spectrum whose one-loop triangle and higher-polygon anomalies cancel against the classical variation of a corresponding Green-Schwarz counterterm on Y.

8 Phenomenological Signatures and Experimental Tests

8.1 Precision Quantum Optical Visibility Attenuation

In a multi-path quantum interferometer enclosing a spatial 3-chain V_3 bounded by closed paths $\gamma = \partial\Sigma^2$, single-photon holonomies $\oint_\gamma A = \int_{\Sigma^2} F$ acquire a topological phase shift from the current J_s through the field equation $d * F = J_s$:

$$\Delta\phi_s = \int_{V_3} J_s 1001[1]. \tag{29}$$

For an isotropic k-torus compactification of scale Λ, stochastic bulk vacuum fluctuations with zero mean and variance $\langle(\Delta\phi_s)^2\rangle \sim \|\omega(\eta)\|^2\Lambda^{-2k}$ induce an interference visibility suppression [1]:

$$1 - V \approx \frac{1}{2}\langle(\Delta\phi_s)^2\rangle \sim \|\omega(\eta)\|^2\Lambda^{-2k}1001[1]. \tag{30}$$

We propose this scaling as a phenomenological hypothesis testable in ultra-stable optical cavities with phase sensitivities $\delta\phi \sim 10^{-10}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$ [1].

8.2 Muon Anomalous Magnetic Moment and Atomic Parity Violation

At loop level, the topological coupling modifies the electromagnetic propagator, generating a shift in the muon anomalous magnetic moment:

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} \approx \frac{\alpha}{2\pi} C_\mu \left(\frac{m_\mu}{\Lambda}\right)^2 \|\omega(\eta)\|^2 1001[1]. \tag{31}$$

Furthermore, neutral-current mixing induces a shift in the weak charge of Cesium (^{133}Cs):

$$\Delta Q_W(^{133}\mathrm{Cs}) = -2\left[(2Z+N)\Delta\sin^2\theta_W\right] \approx -376\,\Delta_{\mathrm{topo}}(q^2 \approx 0) \tag{32}$$

establishing concrete, correlated experimental tests spanning atomic, optical, and high-energy regimes.

9 Conclusion

We have presented an extended Standard Model framework based on $(4+k)$-dimensional geometry, characteristic topological classes, and fiber pushforward calculus [1]. By embedding electroweak interactions into holomorphic adjoint sheaves $\mathcal{E}_{EW} = \mathcal{O}_Y(\mathrm{ad}(P_{EW})) \in \mathrm{Coh}(Y)$ on a complex analytic fiber bundle $Y \xrightarrow{\pi} X$ and modeling physical electromagnetism via the unbroken photonic line subbundle $\mathcal{L}_\gamma^{\mathrm{ad}} \subset \mathrm{ad}(P_{EW})$, we derived the closed spacetime soul current $J_s = \pi_* \mathcal{J} \in \Omega^3(X)$ and formulated the degree-consistent field equations $dF = 0$ and $d * F = J_s + eJ_{\mathrm{em}}$ [1].

Under the stated geometric postulates, the framework:

1. Topologically realizes a net chiral index of three ($\mathrm{ind}(\mathcal{D}_K) = 3$) for fermion generations via the Atiyah-Singer index theorem on compact spin fibers with $c_1(\mathcal{V}_F) = 0$.

2. Formulates an effective threshold regularization of the electroweak scale via Kaluza-Klein compactification geometry.

3. Preserves exact 4D gauge invariance and recovers classical electrodynamics under the $k = 0$ decoupling limit [1].

4. Provides testable, correlated phenomenological predictions across quantum optical interferometry ($1 - V \sim \|\omega(\eta)\|^2 \Lambda^{-2k}$), $(g-2)_\mu$, and atomic parity violation [1].

Future work will investigate explicit characteristic cohomology classes for specific Calabi-Yau compactifications, determine full chiral fermion spectra for Green-Schwarz anomaly cancellation, and explore embeddings into Grand Unified gauge groups including $SO(10)$ and $E_8 \times E_8$ string compactifications [1, 3].

References

[1] P. De Ceuster, "Photon Soul Continuity: An Unobserved Extension of Maxwell's Equations," Technical Preprint, August 2025.

[2] D. S. Freed, "Dirac charge quantization and generalized differential cohomology," *Surveys in Differential Geometry*, vol. 7, pp. 129–194, 2000.

[3] E. Witten, "Geometric Langlands and the equations of Nahm and Bogomolny," *arXiv preprint arXiv:1602.09021* [hep-th], 2016.

[4] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley, Reading, MA, 1995.