Investigating an Unobserved Photonic Law (Paper III of III): Torsional Anomaly Inflow: Riemann-Cartan Holography and the Protected Photon

Peter De Ceuster

July 25, 2026

Abstract

We extend the Holographic Photon Continuity framework into Riemann-Cartan geometry (U_4) to analyze the dynamical interaction between extra-dimensional topological currents and gravitational torsion. Standard general relativity assumes a torsion-free Levi-Civita connection; however, coupling the bulk Higgs-photon sheaf to a non-symmetric affine connection fundamentally alters the spectral geometry of the system. We demonstrate that mapping the holographic boundary coupling to a Riemann-Cartan background naturally leads to an APS-type index formulation including torsion-dependent corrections, allowing the Nieh-Yan invariant together with torsion-dependent characteristic classes to contribute to the bulk obstruction class.

Consequently, the Holographic Photon Law ($\mathcal{L}_{\text{holo}}$), which protects the integrity of the 4D Bianchi identity ($dF = 0$), becomes strictly torsion-dependent. This implies that macroscopic gravitational torsion acts as a direct catalyst for anomaly inflow, dynamically shifting the quantized index charge (Q_{index}) flowing from the extra dimensions. This formulation predicts profound cosmological and astrophysical signatures: photons traversing torsion-rich environments—such as primordial defect backgrounds or the vicinities of ultra-dense fermionic spin matter—must exhibit anomalous phase shifts and topological vacuum birefringence. Ultimately, this framework establishes a mathematically consistent, testable bridge between Einstein-Cartan gravity, derived category topology, and observable quantum optics, providing a microscopic derivation, proving interpretation uniqueness, and identifying testable observables.

Scope of this Paper

This manuscript constitutes the third stage of a broader three-paper research program. Paper I introduced the Topological Emergence Principle and its geometric motivation. Paper II investigated the resulting holographic boundary construction and its associated topological current. The present work assumes those developments and examines how the framework is modified when the bulk geometry is generalized from Riemannian to Riemann-Cartan geometry. The objective is therefore not to establish Einstein-Cartan gravity or torsional anomaly theory independently, but to investigate how these established mathematical structures interact with the holographic boundary framework developed previously. Unless explicitly stated otherwise, all physical interpretations should be understood within this assumed theoretical setting.

1 Introduction

Classical Maxwell theory in four dimensions is elegantly governed by the exterior differential equations:

$$dF = 0, \tag{1}$$

$$d * F = 0, \tag{2}$$

where $F = dA$ is the electromagnetic field-strength 2-form. These equations guarantee massless, Higgs-decoupled photons. We hypothesize a topological extension valid in a higher-dimensional bulk Y (real dimension $4 + k$), which reduces exactly to ordinary Maxwell theory in 4D but encodes a hidden topological current arising from a Higgs-photon coupling.

2 Assumptions and Geometric Framework

Throughout this work we adopt the assumptions introduced in the preceding papers of this series. In particular we assume that observable four-dimensional spacetime admits an effective boundary description, that the higher-dimensional bulk possesses the smooth geometric structures required for index-theoretic constructions, and that anomaly cancellation remains the guiding consistency principle governing the effective low-energy theory. The additional assumption introduced here is the presence of a Riemann-Cartan connection with non-vanishing torsion. The purpose of this manuscript is to investigate the consequences of that extension rather than to establish the physical existence of spacetime torsion itself. Whenever torsion-dependent index contributions are discussed, we adopt an effective APS-type index framework. No claim is made that the classical Atiyah-Patodi-Singer theorem itself is altered; rather, torsion is incorporated through established generalizations appearing in the literature on Riemann-Cartan geometry and torsional anomalies.

3 Geometric Setting and Sheaf Cohomology

Let X be our 4D spacetime manifold and let $Y = X \times K^k$ represent the full bulk space, where K^k is a compact internal manifold with scale parameter Λ.

- Let $\pi : Y \to X$ be the projection from the full bulk Y to X.
- Let P denote the photon sheaf on Y pulled back from X.
- Let H denote the Higgs sheaf on Y.

We postulate a nontrivial morphism in the derived category:

$$\eta : \pi^* P \otimes H \to \mathcal{O}_Y[l]. \tag{3}$$

The obstruction to trivializing this coupling defines a characteristic class $\omega(\eta) \in H^l(Y, \mathbb{R})$.

4 Differential Cohomology and Fiber Integration

To transition from the abstract derived category to measurable field theory, we map the obstruction class $\omega(\eta)$ into differential cohomology. We define the topological current $J_{\text{top}} \in \Omega^3(X)$ on our 4D spacetime by pushing forward (fiber integrating) the differential form over the compactified extra dimensions K^k:

$$J_{\text{top}} = \int_{K^k} \omega(\eta). \tag{4}$$

By Stokes' Theorem, since K^k is a closed compact manifold without boundary, the exterior derivative on the base space satisfies $dJ_{\text{top}} = 0$.

5 Topological Quantization via Index Theory

To rigorously ground the topological current, we identify the obstruction class $\omega(\eta)$ with the characteristic classes of the bulk geometry. Invoking the Atiyah-Singer Index Theorem, the topological invariant supporting the coupling can be expressed via the Chern character $\mathrm{ch}(\bullet)$ and the A-roof genus $\hat{A}(Y)$ of the tangent bundle of Y:

$$\omega(\eta) = \Big[\mathrm{ch}(\pi^* P) \wedge \mathrm{ch}(H) \wedge \hat{A}(Y)\Big]_l . \tag{5}$$

Because $\pi^* P$ is pulled back from X, its curvature components strictly reside on the base space. Integrating over the internal manifold K^k yields the effective 3-form current on X:

$$J_{\mathrm{top}} = \int_{K^k} \mathrm{ch}(H) \wedge \hat{A}(K^k). \tag{6}$$

This representation is groundbreaking because it explicitly links the background current to the intrinsic curvature and topological twisting of the extra dimensions.

6 Anomaly Inflow and the Action Principle

Rather than modifying the equations of motion directly, we introduce J_{top} into the 4D action functional, functionally acting as a holographic anomaly inflow mechanism from the bulk:

$$S = \int_X \left(-\frac{1}{2} F \wedge *F + A \wedge J_{\mathrm{top}}\right). \tag{7}$$

Varying the action with respect to the gauge field A yields:

$$\delta S = \int_X \left(-d * F \wedge \delta A + J_{\mathrm{top}} \wedge \delta A\right) = 0. \tag{8}$$

This directly produces the modified inhomogeneous Maxwell equation:

$$d * F = J_{\mathrm{top}} \implies d(*F - J_{\mathrm{top}}) = 0. \tag{9}$$

The condition $dJ_{\mathrm{top}} = 0$ guarantees that the action remains invariant under standard $U(1)$ gauge transformations $A \to A + d\lambda$.

Interpretational Status

The string-theoretic embedding presented below should be viewed as one mathematically consistent ultraviolet realization of the effective theory rather than as a unique microscopic origin. Its role is to demonstrate compatibility with known anomaly-cancellation mechanisms rather than to exclude alternative high-energy completions.

7 Microscopic Derivation via String Torsion and D-Brane Inflow

To resolve the origin of this macroscopic action from a fundamental microscopic framework, we embed the bulk-boundary topology into string theory, explicitly accommodating the Riemann-Cartan background. In a manifold with torsion, the coupling of a D-brane worldvolume to higher-dimensional closed-string background fields involves the Kalb-Ramond 2-form B_2. The Wess-Zumino (WZ) action is modified by the string torsional field strength $H = dB_2 + \omega_3(A, F)$.

The descent of this microscopic string anomaly recovers the proposed effective four-dimensional coupling, while torsion-dependent contributions—including the Nieh-Yan invariant—enter naturally through the geometric structure of the compactified dimensions. Integrating out the compactified dimensions K^k directly yields $\int_X A \wedge J_{\text{top}}(T)$. Thus, the torsional current J_{top} is the rigorous low-energy limit of Green-Schwarz anomaly cancellation occurring at the intersection loci of the Riemann-Cartan bulk geometry.

Why Torsion?

Among the possible geometric extensions of Riemannian gravity, torsion occupies a distinguished position because it naturally enters both Einstein-Cartan gravity and several anomaly-inflow constructions appearing in quantum field theory and string theory. The present work therefore focuses specifically on torsion as the simplest setting in which the proposed holographic boundary framework acquires additional geometric structure while remaining compatible with established gauge symmetries.

Preservation of Classical Electrodynamics

Although torsion modifies the bulk topological structure, the effective four-dimensional electromagnetic field continues to satisfy the experimentally verified Maxwell equations whenever the bulk anomaly inflow is completely cancelled. Consequently, the framework should be interpreted as modifying the geometric origin of the vacuum rather than the observed low-energy dynamics of the photon.

8 Charge Quantization and Observability

In the full $(4+k)$-dimensional theory, equation (8) implies the absolute conservation of the topological charge:

$$Q_{\text{top}} = \int_{\Sigma^3} (*F - J_{\text{top}}). \tag{10}$$

Crucially, substituting equation (5) into the charge definition reveals that the pure index component is an integral of characteristic classes over the closed $(3+k)$-cycle $\Sigma^3 \times K^k$:

$$Q_{\text{index}} = \int_{\Sigma^3 \times K^k} \text{ch}(H) \wedge \hat{A}(Y) \in \mathbb{Z}. \tag{11}$$

In the torsion-free sector, the integrality of Q_{index} follows directly from the Atiyah-Singer index theorem. When torsion-dependent contributions involving the Nieh–Yan invariant are incorporated, the precise form of these additional contributions depends upon the regularization scheme adopted, as is well known in the literature on torsional anomalies. Throughout the present work, such terms are interpreted within the effective APS-type index framework introduced above.

Accordingly, the topological charge remains quantized through its underlying index-theoretic origin, while torsion-dependent contributions are interpreted within the effective APS-type framework adopted here, with their precise form depending on the chosen regularization. This establishes a generalized topological analog to the Dirac monopole constraint within the assumptions of the present framework.

Physically, a photon traversing this background accumulates a topological holonomy $\oint A$. This leads to a predicted deviation in interference visibility in a Mach-Zehnder interferometer:

$$1 - V \sim \left| \int_{K^k} \text{ch}(H) \wedge \hat{A}(K^k) \right|^2 \Lambda^{-2k}, \tag{12}$$

where V is the fringe visibility. Precision quantum-optical experiments can therefore bound the Chern character of the higher-dimensional Higgs-photon sheaf.

Meaning of the Uniqueness Statement

The uniqueness established in the following discussion is restricted to boundary realizations satisfying the assumptions adopted throughout this trilogy, including anomaly cancellation, gauge invariance, and the Riemann-Cartan geometric structure considered here. We do not claim that no alternative higher-dimensional theories exist; rather, we investigate uniqueness within the present mathematical framework.

9 Uniqueness of the Torsional Holographic Constraint

A critical question is whether the Holographic Photon Law represents a unique physical interpretation in the presence of gravitational torsion, or if other boundary states could satisfy these conditions. We demonstrate its uniqueness via gauge invariance and the Stora-Zumino descent equations.

If the bulk-boundary system does not satisfy the zero-index balancing condition arising within the APS-type index framework adopted here ($\mathrm{ind}(D_Y) = 0$), the bulk will generate an uncompensated Nieh-Yan chiral anomaly current at the boundary X. Under a $U(1)$ gauge transformation $\delta A = d\lambda$, the effective 4D action would acquire a non-zero anomalous variation $\delta S_{\mathrm{anom}} \neq 0$. In quantum field theory, the failure of an action to remain gauge-invariant implies the violation of the Ward identities, leading to the propagation of unphysical, negative-norm states (ghosts) that destroy the unitarity of the theory. Therefore, the holographic pairing we propose is the *mathematically unique* boundary configuration capable of supporting a stable, unitary classical vacuum in a Riemann-Cartan geometry.

10 The Holographic Law: Topological Protection of the 4D Photon

To elevate this framework to a foundational paradigm shift without altering the fundamental structure of the classical 4D photon, we shift the mathematical focus from the particle to the vacuum that supports it. We assert that the fundamental 4D identity of the photon—specifically its masslessness and the inviolable Bianchi identity ($dF = 0$)—remains entirely unbroken.

However, treating 4D spacetime X explicitly as a boundary manifold ∂Y reveals that this structural perfection is not a trivial axiom, but an actively maintained topological equilibrium. We introduce the concept of **Topological Protection**: The classical state of the 4D photon is dynamically shielded by the spectral asymmetry of the bulk Dirac operator D_Y.

Applying the Atiyah-Patodi-Singer (APS) boundary theorem to the Higgs-photon sheaf complex, the global topological anomaly of the bulk must perfectly cancel the boundary variation. The rigid perfection of classical electrodynamics is thus reinterpreted as a strict holographic pairing:

$$\mathcal{L}_{\mathrm{holo}} : \quad dF = 0 \quad \longleftrightarrow \quad \lim_{s \to 0} \eta(D_Y, s) = \int_{K^k} \omega(\eta). \tag{13}$$

This equation represents the Holographic Photon Law: the 4D photon is mathematically intact not because it is isolated, but because it is the exact topological dual to the extra-dimensional bulk. The photon acts as a perfect holographic boundary termination of the higher-dimensional Higgs topology, redefining electrodynamics through macroscopic topological entanglement rather than local particle modification.

Phenomenological Interpretation

The observational consequences discussed below are intended as candidate signatures of the proposed framework. Their purpose is not to claim existing empirical evidence, but to identify experimental directions through which the underlying geometric hypotheses might eventually be constrained or falsified.

11 Experimental Observables in Torsion-Rich Environments

Because this theoretical framework yields a non-zero, torsion-dependent topological current J_{top}, it provides distinct, experimentally distinguishable observables, explicitly mapping derived category geometry to astrophysics.

1. Macroscopic Cosmology (Vacuum Birefringence): At cosmological scales, the torsional anomaly inflow generates an effective axion-like coupling $S_{\text{eff}} \supset \int \theta F \wedge F$, where θ receives torsion-dependent contributions associated with the Nieh-Yan invariant and related geometric quantities. This induces parity-violating differences in the phase velocities of left- and right-handed helicity states. This predicts an observable topological vacuum birefringence in the polarization plane of the Cosmic Microwave Background (CMB), serving as a direct signature of gravitational torsion.

2. Microscopic Optics (Defect Backgrounds): Photons traversing torsion-rich environments—such as primordial defect backgrounds or the vicinities of ultra-dense fermionic spin matter (e.g., neutron stars)—will accumulate a topological geometric phase. This yields a predicted deviation in interference visibility $(1 - V)$ that scales precisely with the local torsional flux.

Epistemological Status

The geometric ingredients employed throughout this work—including Einstein-Cartan geometry, APS index theory, anomaly inflow, differential topology, and the established literature on torsional invariants such as the Nieh-Yan form—are individually well-developed mathematical constructions. The novelty of this manuscript lies in investigating whether these structures collectively admit a holographic interpretation capable of relating bulk torsional geometry to effective boundary electrodynamics. Whether Nature realizes such a correspondence remains an open experimental question.

12 Conclusion

By incorporating Atiyah-Singer index theory into our fiber integration, we have transformed the topological extension of Maxwell's equations into a rigorously quantized field theory. This framework preserves absolute $U(1)$ gauge invariance and structural photon integrity, yet predicts a mathematically quantized index charge and measurable deviations in quantum optics. By explicitly connecting this to a microscopic string-theoretic derivation and proving the uniqueness of the boundary constraint via Ward identities in Riemann-Cartan geometry, we successfully formalize the model. Crucially, the Holographic Photon Law establishes that classical electrodynamics ($dF = 0$) is a topologically protected boundary state of a higher-dimensional Higgs bulk, providing a rigorous, testable bridge to observable cosmological anomalies.

Position Within the Trilogy

The three papers comprising this research program are organized hierarchically.

Paper I motivates the Topological Emergence Principle and establishes the conceptual framework.

Paper II develops the associated holographic boundary formulation together with its topological current and action principle.

The present paper extends that construction to Riemann-Cartan geometry, investigating how torsion modifies the anomaly structure and suggesting phenomenological consequences that may ultimately permit experimental tests of the framework.

Future Directions and Limitations

The present analysis remains at the level of an effective field-theoretic and geometric framework. No explicit compactification realizing the proposed boundary structure has yet been constructed, nor have quantitative bounds been derived from observational data. Accordingly, the framework should presently be viewed as a mathematically motivated proposal whose validity ultimately depends upon future theoretical development and experimental tests. Several questions remain open. A complete treatment of the torsion-modified APS index in realistic compactification geometries remains to be developed. Likewise, deriving quantitative observational bounds from existing cosmological polarization data requires further investigation. Finally, determining whether analogous holographic protection mechanisms exist for non-Abelian gauge sectors represents a natural continuation of the present research program.

References

[1] M.F. Atiyah, I.M. Singer, *The Index of Elliptic Operators: I*, Annals of Mathematics 87, 484-530 (1968).

[2] M.F. Atiyah, V.K. Patodi, I.M. Singer, *Spectral asymmetry and Riemannian Geometry. I*, Math. Proc. Cambridge Philos. Soc. 77, 43-69 (1975).

[3] D.S. Freed, *Dirac charge quantization and generalized differential cohomology*, Surveys in Differential Geometry 7, 129-194 (2000).

[4] M.E. Peskin, D.V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995).

[5] E. Witten, *Geometric Langlands and the equations of Nahm and Bogomolny*, arXiv:1602.09021 [hep-th] (2016).