Investigating an Unobserved Photonic Law (Paper I of III): Holographic Boundary States from Optics to String Theory

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Abstract

Throughout the history of physics, deeper theories have not invalidated successful lower-energy laws; rather, they have explained their origin. Newtonian gravity emerged from general relativity as a limiting case. Thermodynamics emerged from statistical mechanics. Hydrodynamics emerged from molecular dynamics. Likewise, if classical electrodynamics admits a higher-dimensional topological origin, Maxwell's equations would not be replaced but understood as the effective boundary manifestation of a deeper organizing principle. The consistency of classical electromagnetism, defined by the Bianchi identity ($dF = 0$), is traditionally treated as an axiomatic property of four-dimensional spacetime. However, altering Maxwell's equations to reflect deeper topological realities should not be shied away from, provided it is approached with mathematical care and precisely built around existing symmetries. espite the remarkable success of classical electromagnetism, the geometric origin of gauge consistency remains an active topic of theoretical investigation. In this manuscript—the first in a comprehensive set of three papers—we present a theoretical framework investigating an unobserved law of nature that is fundamentally photonic in character. We explore the possibility that the structural integrity of the massless 4D gauge field is a topologically constrained boundary state. By mapping derived category topology into differential K-theory, we formulate a *Holographic Photon Ansatz* ($\mathcal{L}_{\text{holo}}$). This proposes that the classical gauge field acts as a topological dual to a higher-dimensional Higgs-photon bulk, constrained by the spectral asymmetry (Atiyah-Patodi-Singer η-invariant) of the extra dimensions.

This specific work is structured internally into two main parts. In Part I, we outline the mathematical foundations. Assuming compact Kähler manifolds, we formalize the bulk-boundary coupling via Ext-group extension classes. By elevating to differential K-theory, we capture the extension data within the differential Chern character and utilize the Grothendieck-Riemann-Roch theorem to derive a topological anomaly current. We then explicitly link the Atiyah-Patodi-Singer theorem to the Bianchi identity via the Callan-Harvey anomaly inflow mechanism, carefully laying the groundwork for this unobserved photonic law. By postulating a zero-index vacuum condition ($\text{ind}(D_Y) = 0$), we demonstrate that boundary gauge consistency is enforced by neutralizing bulk anomaly inflow.

In Part II, we explore phenomenological applications and open research programs. We utilize the Stora-Zumino descent equations to preserve $U(1)$ gauge invariance against Nieh-Yan torsional inflow. We embed this framework into a warped Kaluza-Klein compactification, defining the Neumann boundary conditions required to localize the massless zero-mode. For quantum optics, we propose a phenomenological decoherence model for topological phase accumulation. Finally, we conjecture a modification to the Witten-Veneziano mass formula for $SU(N)$ theories, proposing that the dynamic generation of the mass gap (Λ_{QCD}) may be linked to bulk spectral asymmetry.

Contents

Preface: Philosophy of Discovery

A Note on Scientific Method

Throughout this manuscript we deliberately distinguish between established mathematical structure and physical interpretation. Every theorem, construction, and geometric identity employed herein—including differential K-theory, Atiyah-Patodi-Singer index theory, anomaly inflow, and Grothendieck-Riemann-Roch—is standard mathematics. What is new is the proposal that these independent structures may collectively admit an alternative physical interpretation.

We therefore use the expression *unobserved law* in the precise sense of theoretical physics: a candidate organizing principle whose mathematical consistency motivates experimental investigation. No claim of experimental verification is made. Rather, the purpose of this work is to investigate whether known geometric structures admit a unified interpretation that naturally reproduces the observed consistency of classical electromagnetism.

The framework developed here introduces no new fundamental particles, no arbitrary coupling constants, and no phenomenological correction terms. Instead, every construction begins from established mathematical structures already employed independently throughout gauge theory, topology, and string theory. The novelty lies exclusively in proposing that these structures admit a common physical interpretation.

Why Search for a Deeper Photonic Law?

Maxwell's equations are experimentally complete within their established domain. However, completion does not imply fundamentality. Newtonian gravity remained correct within its low-energy, weak-field domain despite being superseded by general relativity. Classical thermodynamics remained correct after statistical mechanics explained it microscopically. Hydrodynamics remained macroscopically correct even after molecular theory was established.

Likewise, Maxwell's equations could remain exactly valid in 4D while admitting a much deeper topological origin. Historically, numerous developments in theoretical physics originated as mathematical consistency conditions before experimental confirmation. Gauge invariance, general covariance, non-Abelian Yang-Mills theory, spontaneous symmetry breaking, anomaly cancellation, and holography all existed first as mathematical necessities. The present work follows the same philosophy: mathematical consistency is treated as the primary guide for constructing candidate physical principles. Our approach means, we are no longer holding a flashlight in front of the sun. The sheaf comes first.

Part I

Mathematical Foundations

Chapter 1

The Holographic Boundary and Differential K-Theory

1.1 Introduction to the Boundary State

Classical Maxwell theory relies on the exterior differential equations $dF = 0$ (the Bianchi identity) and $d * F = 0$. While historically treated as rigid axioms, modifying these equations is a necessary step to uncover new fundamental physics, provided the modifications are mathematically protected. We investigate the premise that observable 4D spacetime functions explicitly as a boundary $X = \partial Y$ of a higher-dimensional bulk Y, concealing an unobserved law of nature that is photonic in character. In this framework, the structure of the gauge field is topologically anchored to an anomaly current arising from a bulk Higgs-photon coupling.

Definition (Photonic Law): Within this work, we use the term *photonic law* to denote a higher-dimensional topological constraint whose four-dimensional manifestation reproduces the observed structure of classical electromagnetism. This terminology should therefore not be interpreted as replacing Maxwell's equations, but rather as proposing an underlying geometric principle from which the classical equations emerge as an effective boundary description.

Foundational Hypothesis (Topological Emergence Principle). We propose that every anomaly-free classical gauge sector admits an equivalent geometric interpretation as the boundary state of a higher-dimensional topological bulk satisfying a vanishing net Atiyah–Patodi–Singer (APS) index,

$$\mathrm{ind}(D_Y) = 0.$$

Within this framework, classical gauge consistency is interpreted not as an isolated four-dimensional axiom, but as the effective boundary manifestation of a globally consistent higher-dimensional topology. This hypothesis serves as the central organizing principle of the present work and motivates the mathematical developments explored throughout this manuscript. At present it should be regarded as a physically motivated conjectural principle rather than a proven theorem.

The framework proposed here should therefore not be viewed as a modification of Maxwell theory in its experimentally verified domain. Rather, it proposes an alternative ontological origin for the Bianchi identity itself. Observable four-dimensional electrodynamics remains unchanged whenever the topological consistency conditions of the higher-dimensional bulk are satisfied.

To explicitly delineate our foundational theorems from our physical proposals, we provide a "Conjectural Ladder" (Table 1.1).

Level	**Status**
Differential K-theory	Established
APS Index Theorem	Established
Callan-Harvey inflow	Established
Grothendieck-Riemann-Roch	Established
Boundary interpretation	Proposed
Holographic Photon Ansatz	Conjecture
Optical phenomenology	Speculative
Experimental law	Open question

Table 1.1: The Conjectural Ladder outlining the epistemological status of the framework.

1.2 Sheaf Morphisms and Exact Sequences

Let X and Y be smooth, compact Kähler manifolds, with $Y = X \times K^k$ and $\pi : Y \to X$ being a proper morphism. Let P denote the photon sheaf and H the Higgs sheaf on Y. We formalize the coupling via a derived category extension class:

$$\varepsilon \in \mathrm{Ext}^1_Y(\pi^* P, H) \cong H^1(Y, \mathcal{H}om(\pi^* P, H)). \tag{1.1}$$

Derived categories are introduced not because they are mathematically fashionable, but because extension classes naturally encode non-trivial bulk-boundary couplings. They therefore provide a language in which higher-dimensional interaction data may survive dimensional reduction while remaining invisible to ordinary cohomology. This class classifies the short exact sequence:

$$0 \to H \to E \to \pi^* P \to 0, \tag{1.2}$$

where E represents the bound state of the coupled system in the bulk.

1.3 Differential Chern Characters and the GRR Bridge

While topological K-theory is blind to the extension class—yielding only the additive relation $[E] = [H] + [\pi^* P]$—the physical gauge fields are sensitive to the connection ∇_E. We must therefore elevate our analysis to *differential K-theory* (in the sense of Hopkins and Singer).

The extension class ε acts as an obstruction to a flat splitting, inducing a non-trivial connection structure on E. The differential Chern character $\hat{\mathrm{ch}}(E, \nabla_E)$ captures this specific extension data. To map this to the base manifold X, we apply the Grothendieck-Riemann-Roch (GRR) theorem to the projection π:

$$\hat{\mathrm{ch}}(\pi_! E) = \pi_* \left(\hat{\mathrm{ch}}(E, \nabla_E) \wedge \mathrm{Td}(Y/X) \right), \tag{1.3}$$

where Td reduces to the A-roof genus $\hat{A}$ for spin manifolds. Integrating over the fibers of K^k, we arrive at a very curious equation. We define the topological anomaly current on the boundary, which is strictly dependent on the extension data ε:

$$J_{\mathrm{top}}(\varepsilon) = \int_{K^k} \hat{\mathrm{ch}}(E, \nabla_E) \wedge \hat{A}(K^k). \tag{1.4}$$

Chapter 2

The APS Ansatz and the Bianchi Identity

2.1 Anomaly Inflow and Magnetic Sources

Both information theory and the Langlands program are modern theorems. Both theories were born because their authors identified anomalies. There is a deviation from the standard. To link this mathematical structure to the physical Bianchi identity without breaking foundational consistency, we invoke the Callan-Harvey anomaly inflow mechanism. In a manifold with boundary, a chiral anomaly in the bulk generates a boundary current. If the bulk fields exhibit a non-zero chiral index, the anomaly inflow acts as a magnetic topological defect on the boundary X.

Revealing this unobserved photonic law requires a careful modification of the effective boundary Bianchi identity, which is now sourced by the anomaly current:

$$dF = *J_{\text{anom}}. \tag{2.1}$$

Thus, classical gauge consistency ($dF = 0$) can only be maintained if the net anomaly inflow J_{anom} from the bulk is strictly zero.

The vanishing of the effective anomaly current should not be interpreted as physical triviality. In many areas of theoretical physics—including gauge anomalies, topological phases of matter, and string compactifications—the absence of an observable effect reflects an underlying cancellation between deeper geometric structures. The classical Bianchi identity may therefore encode hidden information regarding the topology of the higher-dimensional bulk, even when no explicit source term survives in the effective four-dimensional description. This demonstrates how a higher-dimensional topological framework can consistently reproduce Maxwell's equations as its effective boundary description. There is NO further consequence; Maxwell's classical law does indeed remain intact.

2.2 The Holographic Vacuum Ansatz

The Atiyah-Patodi-Singer (APS) index theorem for the bulk Dirac operator D_Y states:

$$\operatorname{ind}(D_Y) = \int_Y \operatorname{ch}(E) \wedge \hat{A}(Y) - \frac{\eta(D_X) + h}{2}. \tag{2.2}$$

Because $\mathrm{ind}(D_Y)$ dictates the net chiral anomaly pumped into the boundary, we propose the **Holographic Vacuum Ansatz**: A stable, defect-free 4D classical vacuum requires that the bulk-boundary system possesses zero net index:

$$\mathrm{ind}(D_Y) = 0. \tag{2.3}$$

Setting the index to zero dynamically forces the boundary spectral asymmetry to exactly balance the bulk characteristic classes:

$$\frac{\eta(D_X) + h}{2} = \int_Y \mathrm{ch}(E) \wedge \hat{A}(Y). \tag{2.4}$$

This establishes the *Holographic Photon Ansatz* ($\mathcal{L}_{\mathrm{holo}}$), the mathematical core of the proposed unobserved law. The classical gauge field remains structurally consistent ($dF = 0$) because satisfying the zero-index condition neutralizes the Callan-Harvey anomaly inflow, forbidding the creation of magnetic topological defects on the boundary.

Part II

Phenomenological Applications

Chapter 3

Torsional Anomaly Inflow and Descent Equations

3.1 Riemann-Cartan Geometry and Nieh-Yan Inflow

Both Witten and Weyl have worked with gauge transformations. In a Riemann-Cartan manifold (U_4), the anomaly inflow acquires a contribution from the Nieh-Yan torsional invariant N_{NY}. Coupling macroscopic torsion to the gauge field ($A \wedge J_{\text{top}}$) introduces an anomalous boundary variation under gauge transformations $\delta_\lambda A = d\lambda$, taking the form $\delta S_{\text{anom}} = \int_X \lambda \, dJ_{\text{top}}$.

3.2 Green-Schwarz Cancellation via Stora-Zumino Descent

To carefully preserve 4D $U(1)$ gauge invariance in the presence of this new photonic law, this variation must be canceled by a bulk Kalb-Ramond 2-form B_2. The generalized field strength is $\mathcal{H}_3 = dB_2 - \omega_3(A, F)$, where ω_3 is the Chern-Simons 3-form. Following the Stora-Zumino descent equations, where $d\omega_2^1 = \delta_\lambda \omega_3$, the bulk B_2 field must transform as:

$$\delta B_2 = \omega_2^1(\lambda, A) = \text{Tr}(\lambda dA). \tag{3.1}$$

This Green-Schwarz mechanism ensures that the variation of the bulk B_2 absorbs the boundary torsional anomaly, demonstrating that changes to the classical vacuum equations are symmetry-protected. In other words, anomaly-free.

Chapter 4

Warped Geometries and Zero-Mode Localization

4.1 Boundary Conditions for the Kaluza-Klein Tower

To suppress the Kaluza-Klein (KK) tower, we embed the theory into a warped Randall-Sundrum compactification metric. Localization of the gauge zero-mode is not automatic. By imposing Neumann boundary conditions ($\partial_y A_\mu = 0$) on the IR boundary brane and scaling the bulk gauge kinetic term, the equation of motion yields a flat wavefunction profile for the $n = 0$ zero-mode. The massive KK modes $A^{(n)}$ are localized near the UV brane, raising their effective masses toward the Planck scale.

Chapter 5

Optical Signatures and Phenomenological Bounds

5.1 A Phenomenological Model for Decoherence

Let us say there is a photon propagating through a region permeated by J_{top} and the photon accumulates a non-local geometric phase. In the context of massive Mach-Zehnder interferometry, we propose an empirical phenomenological model for topological phase accumulation to detect this unobserved law. Modeled on standard open-quantum-system decoherence scaling, we construct a phenomenological ansatz where fringe visibility V decays exponentially with the curious topological interaction:

$$1 - V = \exp\left(-\frac{1}{\Lambda^{2k}}\left|\int_X J_{\text{top}} \wedge A\right|^2\right). \tag{5.1}$$

Testing this phenomenological bound offers a potential pathway to constrain the compactification scale Λ.

Chapter 6

Conjectures on Dynamic Mass Gaps in $SU(N)$

6.1 Instanton Gas Dynamics and the Topological Index

We explore generalizations to $SU(N)$ Yang-Mills theories. In the Euclidean path integral, the QCD vacuum is weighted by a topological phase. We conjecture that the topological charge density $q(x) = \frac{1}{32\pi^2}\mathrm{Tr}(F \wedge F)$ is constrained by the bulk inflow via the ansatz: $q(x) \propto *dJ_{\mathrm{top}}$.

To correctly model the dynamics of this topological sector, we extend our framework by treating the instanton ensemble as a quantum gas. Drawing upon Villani's entropy methods and the unification of classical gas kinetics with quantum transport properties [5], we postulate that the instanton gas evolution is governed by a quantum-corrected Boltzmann equation [5]. In this formulation, the collision operator incorporates bosonic statistics [5], mirroring the physics of the Bose-Hubbard model [5].

Crucially, we apply Villani's principle of hypocoercivity, extended to the quantum topological vacuum [5]. Assuming the linearized quantum collision operator for the instanton gas admits a spectral gap [5], the system must exhibit exponential relaxation toward its equilibrium state [5]. This hypocoercive relaxation is mathematically bounded by $||f(t) - f_{BE}||_{L^2_{x,v}} \leq Ce^{-\lambda t}||f(0) - f_{BE}||_{L^2_{x,v}}$ [5]. Within our holographic framework, this implies that the dynamic stabilization of the Yang-Mills topological charge density undergoes a rapid relaxation, bridging microscopic bosonic correlations with macroscopic astrophysical observables [5].

6.2 A Holographic Ansatz for Witten-Veneziano

If this ansatz holds, we conjecture a modification to the Witten-Veneziano formula, proposing that the topological susceptibility χ_t is related to the bulk spectral asymmetry:

$$\chi_t = \int d^4x \langle q(x)q(0)\rangle \sim \frac{1}{\mathcal{V}}\left(\lim_{s\to 0}\eta(D_Y, s)\right)^2. \tag{6.1}$$

Establishing this derivation precisely from the higher-dimensional path integral remains an open research program.

Chapter 7

Conclusion

This manuscript—the first of a three-part investigation—outlines a research program evaluating 4D gauge fields as topologically constrained boundary states, investigating the possible existence of an unobserved photonic law of nature. By mapping derived category extensions to differential K-theory and invoking Callan-Harvey anomaly inflow, we provide a consistent geometric framework linking the APS theorem to the Bianchi identity, demonstrating that carefully extending Maxwell's equations yields profound topological insights.

Outstanding Problems

The present work leaves several important questions unresolved. First, a complete derivation of the proposed anomaly current from a microscopic higher-dimensional action remains to be constructed. Second, the uniqueness of the holographic interpretation has not been established. Third, identifying experimentally distinguishable observables remains an open problem. These questions define the primary objectives of the subsequent papers in this series.

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