

Čech-Dolbeault Realizations, Green-Schwarz Anomaly Consistency, and Heterotic Embeddings of Topological Photonic Sheaves

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Abstract

We formulate a cohomological construction and string-theoretic embedding framework for higher-dimensional photonic continuity over a ten-dimensional spacetime product manifold $Y = X \times K$, where X is four-dimensional physical spacetime and K is a compact Calabi-Yau threefold ($k = 6$, $\dim_{\mathbb{R}} Y = 10$) [1]. Addressing the vanishing of harmonic 1-forms on strict Calabi-Yau manifolds ($b_1(K) = 0$), we utilize the canonical horizontal-vertical graded decomposition $\Omega^9(Y) = \bigoplus_{p+q=9} \Omega^p(X) \otimes \Omega^q(K)$ [1]. We formulate a closed, integral differential 9-form representative $\mathcal{J} \in \Omega_{\text{cl},\mathbb{Z}}^9(Y)$ whose leading vertical component $\mathcal{J}_{(3,6)} = \omega_3(X) \wedge \nu_K$ (where $\nu_K \in \Omega_{\text{cl},\mathbb{Z}}^6(K)$ is a normalized closed integral representative of the top cohomology class satisfying $\int_K \nu_K = 1$, and $\omega_3(X) \in \Omega_{\text{cl},\mathbb{Z}}^3(X)$ is a closed spacetime 3-form) guarantees an exact, non-trivial fiber pushforward soul current $J_s = \pi_* \mathcal{J} = \int_K \mathcal{J}_{(3,6)} = \omega_3(X) \in \Omega^3(X)$ satisfying $d_X J_s = 0$ [1].

We examine the embedding of this structure into $E_8 \times E_8$ heterotic string compactifications under consistent adjoint trace conventions $\text{Tr} \equiv \text{Tr}_{\mathbf{248}}$ for E_8 and $\text{tr} \equiv \text{tr}_{\text{vector}}$ for Lorentz curvature. In these conventions, the irreducible gauge and gravitational anomalies vanish through exceptional Casimir identities, allowing the ten-dimensional anomaly 12-form to factorize as $I_{12}(\mathcal{F}, \mathcal{R}) = X_4 \wedge X_8$. The classical variation of the Green-Schwarz counterterm $S_{\text{GS}} \propto \int_Y B_2 \wedge X_8$ cancels the resulting factorized one-loop anomaly, providing a consistent ten-dimensional background for the proposed anomaly-inflow interpretation of J_s [1].

Furthermore, evaluating the Atiyah-Singer index on the Tian-Yau quotient Calabi-Yau threefold $K = \bar{K}/\mathbb{Z}_3$ with Euler characteristic $\chi(K) = -18/3 = -6$ under the standard embedding $\mathcal{V}_F \cong TK$, we derive a net chiral index $\text{ind}(\mathcal{D}_K) = n_{27} - n_{\bar{27}} = \frac{1}{2}|\chi(K)| = 3$, matching the net three-family generation count required by the Standard Model modulo vector-like pairs and Wilson-line symmetry breaking. Finally, we compute the leading-order visibility suppression in precision optical interferometry $1 - V \approx \frac{1}{2}\langle(\Delta\phi_s)^2\rangle \sim \|\omega(\eta)\|^2 \Lambda^{-12}$ [1] and quantify correlated loop-level corrections to the effective weak mixing angle $\sin^2 \theta_W^{\text{eff}}$ and the muon anomalous magnetic moment $(g - 2)_\mu$.

1 Introduction

Unifying the Standard Model within higher-dimensional supergravity and superstring compactifications requires addressing both local gauge anomalies and the geometric origin of low-energy symmetries [3, 4]. In preceding foundational work, we proposed the *Photon Soul Continuity* framework, demonstrating that physical electrodynamics can be augmented by a closed, fiber-pushforward 3-form current $J_s = \pi_* \mathcal{J} \in \Omega^3(X)$ originating from a $(4 + k)$ -dimensional bulk manifold Y [1].

While our previous formulations established the degree consistency of the modified Maxwell equations $dF = 0$ and $d * F = J_s + eJ_{\text{em}}$, two critical structural questions remained to be made mathematically explicit [1]:

1. **Cohomological Representation and Vertical Decomposition:** On strict Calabi-Yau threefolds with $SU(3)$ holonomy, the first Betti number vanishes ($b_1(K) = 0$), precluding the existence of non-trivial harmonic 1-forms on the internal manifold. The construction of the closed 9-form $\mathcal{J} \in \Omega^9(Y)$ must therefore be formulated directly via horizontal-vertical bidegrees (p, q) on the product space $X \times K$ to ensure that fiber integration $\pi_* : \Omega^9(Y) \rightarrow \Omega^3(X)$ produces a non-zero, closed 3-form current on physical spacetime [1].
2. **Heterotic Anomaly Matching and Trace Conventions:** The cancellation of ten-dimensional chiral anomalies must be explicitly verified within the factorized Green-Schwarz mechanism of $E_8 \times E_8$ heterotic compactifications, establishing unified trace normalizations and demonstrating how J_s can be interpreted as a low-energy boundary inflow current [1].

In this paper, we resolve both challenges. We specialize the bulk spacetime to the product manifold $Y = X \times K$, where K is a compact Calabi-Yau threefold ($k = 6$), fixing the total dimension to $\dim_{\mathbb{R}} Y = 10$. Using horizontal-vertical differential algebra, we formulate an explicit differential form representative $\mathcal{J} \in \Omega_{\text{cl}, \mathbb{Z}}^9(Y)$ with non-vanishing $(3, 6)$ vertical component [1].

We embed the gauge structure into $E_8 \times E_8$ heterotic compactifications breaking to effective Grand Unified and Standard Model gauge groups. We verify Green-Schwarz factorization under unified trace normalizations, calculate the 3-generation net chiral index $\text{ind}(\mathcal{D}_K) = 3$ on the Tian-Yau quotient Calabi-Yau manifold with $\chi(K) = -6$, and compute the resulting precision phenomenological signatures across optical interferometry, $(g - 2)_\mu$, and atomic parity violation [1].

2 Geometric Fiber Bundle on Calabi-Yau Threefolds

Let X denote four-dimensional physical spacetime equipped with a smooth Lorentzian metric $g_{\mu\nu}$. We define the ten-dimensional bulk spacetime as the direct product manifold:

$$Y = X \times K \tag{1}$$

equipped with the canonical projection $\pi : Y \rightarrow X$, where the fiber K is a compact, connected Calabi-Yau threefold ($\dim_{\mathbb{C}} K = 3$, $\dim_{\mathbb{R}} K = 6$) satisfying $c_1(K) = 0$ and $\text{Hol}(K) = SU(3)$.

Remark 1 (Cohomological Properties of Strict Calabi-Yau Threefolds). *For an irreducible Calabi-Yau threefold K with full $SU(3)$ holonomy, the Hodge numbers satisfy $h^{0,0} = 1$, $h^{1,0} = 0$, and $h^{2,0} = 0$. Consequently, the first Betti number vanishes identically:*

$$b_1(K) = 2h^{1,0}(K) = 0. \tag{2}$$

Thus, K supports no non-trivial harmonic 1-forms. Any non-trivial differential form on $Y = X \times K$ must therefore be constructed from horizontal forms on X and even-degree vertical forms on K .

2.1 Gauge Bundles and Adjoint Photonic Subsheaves

Let $P \rightarrow Y$ be a holomorphic principal $E_8 \times E_8$ bundle over Y . Spontaneous compactification on K breaks the visible E_8 factor to an effective $SO(10)$ and subsequently to the electroweak gauge group $G_{EW} = SU(2)_L \times U(1)_Y$.

Definition 1 (Electroweak Adjoint Sheaf). *Let $\text{ad}(P_{EW}) = P_{EW} \times_{\text{Ad}} (\mathfrak{sl}(2, \mathbb{C}) \oplus \mathbb{C})$ denote the holomorphic adjoint vector bundle on Y . We define:*

1. *The coherent electroweak adjoint sheaf $\mathcal{E}_{EW} = \mathcal{O}_Y(\text{ad}(P_{EW})) \in \text{Coh}(Y)$ [1].*

2. The bulk Higgs sheaf $\mathcal{H} = \mathcal{O}_Y(\mathcal{V}_H) \in \text{Coh}(Y)$, where $\mathcal{V}_H$ transforms in the fundamental representation of $SU(2)_L$ [1].

Following bulk condensation $\langle \mathcal{H} \rangle = v_B \neq 0$, the unbroken generator $Q = T^3 + Y_W$ stabilizes a one-dimensional trivial holomorphic line subbundle $\mathcal{L}_\gamma^{\text{ad}} = Y \times \text{span}_{\mathbb{C}}\{Q\} \subset \text{ad}(P_{EW})$, defining the coherent photonic projection sheaf:

$$\mathcal{E}_\gamma = \mathcal{O}_Y(\mathcal{L}_\gamma^{\text{ad}}) \in \text{Coh}(Y)1001[1]. \quad (3)$$

3 Horizontal-Vertical Decomposition and Cohomological Representatives

To guarantee that the fiber pushforward yields a non-vanishing 3-form on X , we exploit the canonical global bi-grading of the exterior algebra on the product space $Y = X \times K$:

$$\Omega^n(Y) = \bigoplus_{p+q=n} \Omega^{p,q}(X \times K) = \bigoplus_{p+q=n} \Omega^p(X) \otimes \Omega^q(K). \quad (4)$$

Fiber integration along the 6-dimensional compact manifold K acts as a degree-shifting projection $\pi_* : \Omega^{p,q}(X \times K) \rightarrow \Omega^{p+q-6}(X)$, which annihilates all vertical degrees $q < 6$ and integrates top forms:

$$\pi_*(\alpha_p(X) \wedge \beta_q(K)) = \begin{cases} \alpha_p(X) \int_K \beta_6(K), & q = 6, \\ 0, & q < 6. \end{cases} \quad (5)$$

3.1 Construction of the Differential 9-Form $\mathcal{J}$

Let $\nu_K \in \Omega_{\text{cl},\mathbb{Z}}^6(K)$ denote a normalized closed integral differential form representing the fundamental cohomology class of K satisfying $\int_K \nu_K = 1$, and let $\omega_3(X) \in \Omega_{\text{cl},\mathbb{Z}}^3(X)$ denote a closed integral 3-form on physical spacetime representing a non-trivial class in Cheeger-Simons differential cohomology $\widehat{H}^3(X)$ [2].

Geometric Postulate 1 (Cohomological Representative of the Bulk Interaction). *We postulate that the derived extension class $\eta \in \text{Ext}_Y^9(\pi^* \mathcal{E}_\gamma \otimes^{\mathbf{L}} \mathcal{H}, \mathcal{O}_Y)$ is realized in de Rham cohomology by a differential 9-form:*

$$\mathcal{J} = \mathcal{J}_{(3,6)} + \sum_{p>3} \mathcal{J}_{(p,9-p)} \in \Omega_{\text{cl},\mathbb{Z}}^9(Y)1001[1] \quad (6)$$

whose leading $(3,6)$ horizontal-vertical component is given explicitly by:

$$\mathcal{J}_{(3,6)} = \omega_3(X) \wedge \nu_K \quad (7)$$

satisfying $d_Y \mathcal{J}_{(3,6)} = 0$.

Theorem 1 (Closure and Quantization of $\mathcal{J}_{(3,6)}$). *Let $\mathcal{J}_{(3,6)} = \omega_3(X) \wedge \nu_K$. Then:*

$$d_Y \mathcal{J}_{(3,6)} = 01001[1]. \quad (8)$$

Proof. Applying the exterior derivative $d_Y = d_X + d_K$:

$$d_Y \mathcal{J}_{(3,6)} = (d_X \omega_3) \wedge \nu_K + (-1)^3 \omega_3 \wedge (d_K \nu_K). \quad (9)$$

Since $\omega_3(X)$ is closed on spacetime ($d_X \omega_3 = 0$) and ν_K is closed on the Calabi-Yau threefold ($d_K \nu_K = 0$), both terms vanish identically:

$$d_Y \mathcal{J}_{(3,6)} = 0 + 0 = 01001[1]. \quad (10)$$

Because $\omega_3(X)$ and ν_K have integral periods in Cheeger-Simons differential cohomology, $\mathcal{J}_{(3,6)} \in \Omega_{\text{cl},\mathbb{Z}}^9(Y)$ [1, 2]. $\square$

Remark 2 (Connection to Chern-Simons Forms on Parity-Conserving Backgrounds). *On physical electromagnetic backgrounds satisfying the vanishing four-dimensional topological density condition $F \wedge F = 0$ (such as electrostatic, magnetostatic, or plane-wave configurations), the abelian Chern-Simons 3-form $\text{cs}_3(A) = A \wedge F$ satisfies $d_X \text{cs}_3(A) = F \wedge F = 0$, providing an explicit local gauge realization for $\omega_3(X)$.*

4 Fiber Integration and Spacetime Gauge Dynamics

The physical four-dimensional current J_s is generated via pushforward integration over the Calabi-Yau threefold K [1].

Definition 2 (Fiber Pushforward Current). *The physical spacetime 3-form soul current J_s on X is defined by:*

$$J_s := \pi_* \mathcal{J} = \int_K \mathcal{J}_{(3,6)} = \omega_3(X) \int_K \nu_K = \omega_3(X) \in \Omega^3(X)1001[1]. \quad (11)$$

Theorem 2 (Exact Conservation Law on Spacetime). *Because $\partial K = \emptyset$ and $d_Y \mathcal{J} = 0$, the spacetime current J_s is strictly closed:*

$$d_X J_s = 01001[1]. \quad (12)$$

Proof. By the fiberwise Stokes theorem on smooth fiber bundles with boundaryless fiber K ($\partial K = \emptyset$):

$$d_X J_s = d_X(\pi_* \mathcal{J}) = \pi_*(d_Y \mathcal{J}) + (-1)^{\deg(\mathcal{J})-6} \int_{\partial K} \mathcal{J} = \pi_*(0) + 0 = 01001[1]. \quad (13)$$

□

4.1 Modified Maxwell Field Equations and Gauss-Law Charge

The four-dimensional effective field equations for physical electromagnetism are:

$$dF = 01001[1], \quad (14)$$

$$d * F = J_s + e J_{\text{em}}1001[1, 4]. \quad (15)$$

For any compact spatial 3-chain with boundary $\Sigma^3 \subset X$, the enclosed Gauss-law topological charge is:

$$Q_{\text{soul}}(\Sigma^3) = \int_{\Sigma^3} J_s = \int_{\partial \Sigma^3} *F - e \int_{\Sigma^3} J_{\text{em}}1001[1]. \quad (16)$$

5 Three Generations via Quotient Calabi-Yau Threefolds

In ten-dimensional heterotic compactifications, chiral matter fermions originate from the decomposition of the adjoint representation of E_8 under the maximal subgroup $SU(3)_K \times E_6 \subset E_8$:

$$\mathbf{248} \rightarrow (\mathbf{1}, \mathbf{78}) \oplus (\mathbf{3}, \mathbf{27}) \oplus (\bar{\mathbf{3}}, \bar{\mathbf{27}}) \oplus (\mathbf{8}, \mathbf{1}). \quad (17)$$

The 4D chiral generations transform in the fundamental $\mathbf{27}$ representation of E_6 (which contains the standard $\mathbf{16}$ of $SO(10)$ and Standard Model quarks and leptons).

Theorem 3 (Chiral Generation Count on Calabi-Yau Threefolds). *Let K be a Calabi-Yau threefold with tangent bundle TK , and let $\mathcal{V}_F \rightarrow K$ be a holomorphic vector bundle satisfying $c_1(\mathcal{V}_F) = 0$. The net number of chiral generations $N_{\text{gen}} = n_{27} - n_{\bar{27}}$ is given by the Atiyah-Singer index of the internal Dirac operator $\mathcal{D}_K$:*

$$N_{\text{gen}} = \text{ind}(\mathcal{D}_K) = \int_K \hat{A}(TK) \wedge \text{ch}(\mathcal{V}_F). \quad (18)$$

Proof. For a Calabi-Yau threefold, $c_1(K) = 0$, so $\hat{A}(TK) = 1 + \frac{1}{24}c_2(TK)$. With $c_1(\mathcal{V}_F) = 0$, the Chern character expands as $\text{ch}(\mathcal{V}_F) = \text{rk}(\mathcal{V}_F) - c_2(\mathcal{V}_F) + \frac{1}{2}c_3(\mathcal{V}_F)$. The top degree-6 component is:

$$\left[\hat{A}(TK) \wedge \text{ch}(\mathcal{V}_F) \right]_6 = \frac{1}{2}c_3(\mathcal{V}_F). \quad (19)$$

Integrating over the compact manifold K :

$$N_{\text{gen}} = \frac{1}{2} \int_K c_3(\mathcal{V}_F). \quad (20)$$

□

Proposition 1 (Net Three-Generation Spectrum on the Tian-Yau Threefold). *Let $\tilde{K}$ be a complete intersection Calabi-Yau threefold in $\mathbb{P}^3 \times \mathbb{P}^3$ with Euler characteristic $\chi(\tilde{K}) = -18$, admitting a freely acting discrete symmetry group $G = \mathbb{Z}_3$. The smooth quotient manifold $K = \tilde{K}/\mathbb{Z}_3$ possesses Euler characteristic:*

$$\chi(K) = \frac{\chi(\tilde{K})}{|\mathbb{Z}_3|} = \frac{-18}{3} = -6. \quad (21)$$

Under the standard embedding $\mathcal{V}_F \cong TK$, the index theorem yields:

$$N_{\text{gen}} = \frac{1}{2} \int_K c_3(TK) = \frac{1}{2} |\chi(K)| = \frac{1}{2} |-6| = 3. \quad (22)$$

This establishes a net chiral index of three ($n_{27} - n_{\bar{27}} = 3$). Breaking E_6 down to the Standard Model gauge group and lifting vector-like pairs requires subsequent Wilson-line symmetry breaking utilizing the non-trivial fundamental group $\pi_1(K) \cong \mathbb{Z}_3$.

6 Ten-Dimensional Green-Schwarz Anomaly Consistency

In ten spacetime dimensions, chiral fermions induce one-loop hexagon anomalies described by a characteristic 12-form polynomial $I_{12}(\mathcal{F}, \mathcal{R})$.

6.1 Consistent Trace Conventions and Anomaly Factorization

We adopt the standard Green-Schwarz trace conventions: Tr denotes the trace in the adjoint representation **248** of E_8 , while tr denotes the trace in the fundamental vector representation of $SO(1, 9)$.

For the $E_8 \times E_8$ heterotic theory with gauge curvature $\mathcal{F} = \mathcal{F}_1 \oplus \mathcal{F}_2$, the irreducible 12-form anomaly vanishes identically ($I_{12}^{\text{irr}} \equiv 0$) by virtue of the exceptional Lie algebra identities, allowing the polynomial to factorize into the Green-Schwarz product:

$$I_{12}(\mathcal{F}, \mathcal{R}) = X_4 \wedge X_8 \quad (23)$$

where

$$X_4 = \text{tr } \mathcal{R}^2 - \frac{1}{30} \text{Tr } \mathcal{F}^2, \quad (24)$$

$$X_8 = \frac{1}{24} \text{Tr } \mathcal{F}^4 - \frac{1}{7200} (\text{Tr } \mathcal{F}^2)^2 - \frac{1}{240} \text{Tr } \mathcal{F}^2 \text{tr } \mathcal{R}^2 + \frac{1}{8} \text{tr } \mathcal{R}^4 + \frac{1}{32} (\text{tr } \mathcal{R}^2)^2. \quad (25)$$

6.2 Green-Schwarz Mechanism and Anomaly Inflow Hypothesis

The Kalb-Ramond 2-form field B_2 has modified 3-form field strength $H_3 = dB_2 - \frac{\alpha'}{4}(\omega_{3Y} - \omega_{3L})$, satisfying the Bianchi identity:

$$dH_3 = \frac{\alpha'}{4} \left(\text{tr } \mathcal{R}^2 - \frac{1}{30} \text{Tr } \mathcal{F}^2 \right) = \frac{\alpha'}{4} X_4. \quad (26)$$

Under gauge and Lorentz transformations with parameter Λ , the 2-form transforms as $\delta_\Lambda B_2 = \frac{\alpha'}{4}(\omega_{2Y}^1 - \omega_{2L}^1)$.

Theorem 4 (Bulk Anomaly Cancellation). *Under the standard heterotic Green-Schwarz factorization $I_{12} = X_4 \wedge X_8$, the classical tree-level variation of the Green-Schwarz counterterm:*

$$S_{\text{GS}} = \frac{1}{2(2\pi)^5 \alpha'} \int_Y B_2 \wedge X_8 \quad (27)$$

cancels the factorized 1-loop quantum anomaly $\delta_\Lambda \Gamma_{1\text{-loop}} = -\int_Y I_{10}^{(1)}$:

$$\delta_\Lambda (\Gamma_{1\text{-loop}} + S_{\text{GS}}) = 0. \quad (28)$$

Proof. Using the descent relations $I_{12} = dI_{11}^{(0)}$ and $\delta_\Lambda I_{11}^{(0)} = dI_{10}^{(1)}$, the classical variation of S_{GS} under $\delta_\Lambda B_2 = \frac{\alpha'}{4} d^{-1} X_4^{(1)}$ satisfies:

$$\delta_\Lambda S_{\text{GS}} = \frac{1}{2(2\pi)^5 \alpha'} \int_Y \delta_\Lambda B_2 \wedge X_8 = \int_Y I_{10}^{(1)}. \quad (29)$$

Adding $\delta_\Lambda \Gamma_{1\text{-loop}} + \delta_\Lambda S_{\text{GS}} = 0$ confirms local anomaly matching. $\square$

EFT Hypothesis 1 (Soul Current as Anomaly Inflow). *We hypothesize that the physical 3-form current $J_s = \pi_* \mathcal{J}$ corresponds to the fiber pushforward of the dual anomaly current density $\mathcal{J}_{(3,6)} = X_8 \wedge \nu_K$:*

$$J_s = \pi_* \mathcal{J} = \int_K X_8 \wedge \nu_K \in \Omega^3(X) 1001[1]. \quad (30)$$

Under this correspondence, J_s acts as the low-energy boundary inflow current balancing bulk topological anomalies on physical spacetime [1].

7 Effective Action and Precision Phenomenology

Integrating out massive Kaluza-Klein modes of the Calabi-Yau threefold yields the 4D effective action [1].

Definition 3 (Low-Energy Effective Action). *The gauge-invariant effective action is:*

$$S_{\text{eff}}[A, Z] = \int_X \left(-\frac{1}{2} F \wedge *F - \frac{1}{2} Z_2 \wedge *Z_2 - \frac{1}{2} M_Z^2 Z \wedge *Z + A \wedge J_s + \tan \theta_W Z \wedge J_s + eA \wedge J_{\text{em}} + Z \wedge J_Z \right) 1001 \quad (31)$$

7.1 Quantum Optical Visibility Attenuation

In a multi-path quantum interferometer enclosing a spatial 3-chain V_3 , single-photon states acquire a geometric phase shift $\Delta\phi_s = \int_{V_3} J_s$ [1]. For a Calabi-Yau threefold with compactification scale $\Lambda = 1/R$ ($k = 6$), the internal volume scales as $\text{Vol}(K) \sim \Lambda^{-6}$. In the presence of

stochastic bulk vacuum fluctuations with zero mean and variance $\langle(\Delta\phi_s)^2\rangle \sim \|\omega(\eta)\|^2\Lambda^{-12}$, the interference visibility $V = |\langle e^{i\Delta\phi_s} \rangle|$ expands as [1]:

$$1 - V \approx \frac{1}{2}\langle(\Delta\phi_s)^2\rangle = C_V\|\omega(\eta)\|^2\Lambda^{-12}1001[1] \quad (32)$$

where C_V is an $\mathcal{O}(1)$ geometric form factor determined by the intersection numbers of K . Ultra-stable laser interferometers with phase sensitivity $\delta\phi \sim 10^{-10}$ rad/ $\sqrt{\text{Hz}}$ place direct experimental bounds on $\|\omega(\eta)\|$ for $\Lambda \sim 1\text{--}10$ TeV.

7.2 Electroweak Radiative Corrections

Topological neutral-current coupling induces a scale-dependent shift in the effective weak mixing angle:

$$\sin^2\theta_W^{\text{eff}}(q^2) = \sin^2\theta_W^{\text{SM}}(q^2) + \frac{\sin\theta_W \cos\theta_W \tan\theta_W}{M_Z^2 - q^2} \left(\frac{\|\omega(\eta)\|}{\Lambda^6} \right)^2 1001[1]. \quad (33)$$

At loop level, vacuum polarization diagrams generate a shift in the muon anomalous magnetic moment:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \approx \frac{\alpha}{2\pi} C_\mu \left(\frac{m_\mu}{\Lambda} \right)^2 \left(\frac{\|\omega(\eta)\|}{\Lambda^6} \right)^2 1001[1]. \quad (34)$$

These relations establish concrete, correlated experimental tests spanning optical interferometry, $(g-2)_\mu$, and atomic parity violation in ^{133}Cs [1].

8 Conclusion

We have presented a cohomological construction and string-theoretic embedding framework for topological photonic sheaves on ten-dimensional spacetime $Y = X \times K$ ($k = 6$) [1]. By utilizing horizontal-vertical bidegree decompositions $\Omega^{3,6}(X \times K)$, we formulated the closed integral 9-form $\mathcal{J}_{(3,6)} = \omega_3(X) \wedge \nu_K \in \Omega_{\text{cl},\mathbb{Z}}^9(Y)$ and derived its pushforward 3-form soul current $J_s = \pi_*\mathcal{J} = \omega_3(X) \in \Omega^3(X)$ with $d_X J_s = 0$ [1].

Embedding this structure into $E_8 \times E_8$ heterotic compactifications establishes:

1. 10D Green-Schwarz anomaly matching via factorization $I_{12} = X_4 \wedge X_8$ with explicit adjoint trace normalizations, providing a consistent theoretical setting for the proposed anomaly-inflow interpretation of J_s [1].
2. A net chiral generation index $\text{ind}(\mathcal{D}_K) = n_{27} - n_{\overline{27}} = \frac{1}{2}|\chi(K)| = 3$ on the Tian-Yau quotient Calabi-Yau threefold with $\chi(K) = -6$.
3. Concrete, correlated experimental predictions across optical interference visibility ($1-V \sim \Lambda^{-12}$), weak mixing angle shifts, and $(g-2)_\mu$ [1].

This establishes a mathematically grounded foundation for higher-dimensional photonic continuity in unified field theories [1].

References

- [1] P. De Ceuster, “Photon Soul Continuity: An Unobserved Extension of Maxwell’s Equations,” Technical Preprint, August 2025.
- [2] D. S. Freed, “Dirac charge quantization and generalized differential cohomology,” *Surveys in Differential Geometry*, vol. 7, pp. 129–194, 2000.

- [3] E. Witten, “Geometric Langlands and the equations of Nahm and Bogomolny,” *arXiv preprint arXiv:1602.09021* [hep-th], 2016.
- [4] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley, Reading, MA, 1995.