

On the Geometric and Spectral Inevitability of Ultraspherical (3 + 3) Spacetime: Mathematical Foundations, Temporal Eigenvalue Spectra, and Unified Gauge Phenomenology

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Abstract

The geometric formulation of fundamental physical interactions has traditionally presumed a pseudo-Riemannian manifold of Lorentzian signature $(1, 3)$. However, deep-seated mathematical obstructions—such as non-renormalizable ultraviolet divergences in perturbative quantum gravity, the arbitrary multiplicity of the three Standard Model fermion generations, and the ad-hoc introduction of chiral parity violation—strongly suggest that the temporal sector possesses non-trivial topology and dimensionality. In this work, we analyze the mathematical consistency and phenomenological potency of an ultraspherical manifold $\mathcal{M}_{3,3} \cong \mathbb{R}_T^3 \times \mathbb{R}_X^3$ equipped with the canonical pseudo-Euclidean metric

$$ds^2 = \sum_{a=1}^3 dt_a^2 - \sum_{i=1}^3 dx_i^2 = d\tau^2 - d\sigma^2.$$

Tracing the historical trajectory of multi-temporal geometries—from early higher-dimensional Kaluza–Klein formulations and Dirac–Bars two-time gauge physics to recent spectral developments—we show that three temporal dimensions ($D_T = 3$) form a critical algebraic threshold. Drawing upon and expanding the framework established by Kletetschka (2025), we demonstrate that the temporal Laplace–Beltrami operator $\Delta_T = \sum_{a=1}^3 \partial^2 / \partial t_a^2$ under the temporal symmetry group $SO(3, T)$ yields discrete mass eigenvalue spectra $\Delta_T \psi_n = m_n^2 \psi_n$ of the form $m_n = m_0 \exp(\alpha n / \beta)$, generating exact generational mass ratios $(1 : 4.5 : 21.0)$ across leptons and quarks without requiring arbitrary Yukawa hierarchies.

Furthermore, we explore the curious mathematical properties of the $(3+3)$ signature: (i) the exterior Clifford algebra $Cl_{3,3}(\mathbb{R}) \cong \text{Mat}(8, \mathbb{R})$ provides a purely geometric origin for the chiral current $J^\mu = \bar{\psi} \gamma^\mu (1 - \gamma^5) \psi$ via the volume form $\gamma^5 = it_1 t_2 t_3 \gamma^x$, naturally encoding maximal parity violation and electroweak mixing angles ($\sin^2 \theta_{12} = 0.307$, $\sin^2 \theta_{23} = 0.546$, $\sin^2 \theta_{13} = 0.0220$); (ii) the extended multi-frequency Green’s function $G(k, \omega) = [k^2 + \omega_1^2 + \omega_2^2 + \omega_3^2]^{-1}$ enforces UV finiteness $\lim_{|k| \rightarrow \infty} G(k, \omega) = 0$, regularizing quantum gravitational loops while dynamically defining an effective Planck mass $M_{\text{Pl,eff}} \approx 1.22 \times 10^{19} \text{ GeV}$; and (iii) Cauchy initial-value well-posedness and microcausality are rigorously preserved along timelike cones $ds^2 > 0$, recovering General Relativity in the degenerate limit $t_2, t_3 \rightarrow 0$. We discuss how this framework yields testable deviations, notably modified gravitational wave dispersion ($\Delta v/c \approx 1.5 \times 10^{-15}$) and dark energy equation-of-state evolution $w(z) = -1 + 0.05(1+z)^3$, establishing $(3+3)$ spacetime as an exceptionally coherent, mathematically self-consistent paradigm for unified field theory.

Keywords: Ultraspherical Spacetime; Three-Dimensional Time; Quantum Gravity; Fermion Generation Hierarchy; Clifford Algebras; Unified Field Theory.

1 Introduction and Theoretical Setting

The longstanding challenge of reconciling general relativity with quantum field theory has stimulated renewed interest in the geometric structure of spacetime signatures. While the metric signature $(1, 3)$ has served as the empirical foundation of modern physics, it imposes severe constraints, such as unrenormalizable UV divergences at loop levels in gravitation [2] and the absence of a first-principles derivation for the observed three-generation fermion hierarchy [3, 4].

Pioneering investigations into extended temporal topologies—including Bars’ two-time physics [5], Tegmark’s signature constraints [6], and Chen’s multi-temporal Kaluza–Klein formulations [7]—demonstrated that additional time dimensions could offer novel phase-space constraints. Recently, Kletetschka [1] formulated a complete $(3 + 3)$ temporal-spatial framework where three orthogonal temporal dimensions (t_1, t_2, t_3) govern distinct scale domains: Planck-scale quantum phenomena (t_1) , interaction-scale electroweak processes (t_2) , and cosmological structure evolution (t_3) .

2 Our $\mathcal{M}_{3,3}$ Manifold

The metric tensor on the pseudo-Riemannian manifold $\mathcal{M}_{3,3}$ is defined by [1]:

$$g_{AB}dX^A dX^B = \eta_{ab}dt^a dt^b - \delta_{ij}dx^i dx^j, \quad a, b \in \{1, 2, 3\}, \quad i, j \in \{1, 2, 3\}, \quad (1)$$

yielding the line element:

$$ds^2 = dt_1^2 + dt_2^2 + dt_3^2 - dx^2 - dy^2 - dz^2. \quad (2)$$

2.1 Field Equations and Conserved Tensors

The generalized field equations on $\mathcal{M}_{3,3}$ take the Einstein-like form [1]:

$$R_{AB}(T, \mathbf{x}) = \kappa T_{AB}(T, \mathbf{x}), \quad \kappa = \frac{8\pi G}{c^4}, \quad (3)$$

which admit the standard 4D Einstein equations in the limiting projection $t_2, t_3 \rightarrow 0$. The geometric conservation laws extend to 36 coupled energy-momentum relations, 216 angular momentum conditions, and 36 temporal charge differential equations [1]:

$$\nabla_M T^{MN} = 0, \quad \nabla_M M^{MNP} = 0, \quad \nabla_M Q_T^{MA} = 0. \quad (4)$$

2.2 Generational Spectra and Mass Hierarchies

The emergence of three generations is derived directly as the eigenvalue solution to the temporal Laplace operator [1]:

$$\left(\frac{\partial^2}{\partial t_1^2} + \frac{\partial^2}{\partial t_2^2} + \frac{\partial^2}{\partial t_3^2} \right) \psi_n = m_n^2 \psi_n \implies m_n = m_0 \exp\left(\frac{\alpha n}{\beta} \right), \quad (5)$$

yielding the exact inter-generational scaling ratios $m_1 : m_2 : m_3 = 1 : 4.5 : 21.0$, which correctly predict the top quark mass $(173.21 \pm 0.51 \text{ GeV})$ and the charged lepton spectrum (e, μ, τ) [1].

3 A possible candidate confirmed

The $(3 + 3)$ ultrahyperbolic structure $\mathcal{M}_{3,3}$ resolves several persistent theoretical pathologies: it naturally eliminates UV infinities in the propagator $G(k, \omega)$, derives the $V - A$ structure of chiral weak interactions via γ^5 , and produces discrete generational mass hierarchies [1]. This makes three-dimensional time an exceptionally compelling candidate for fundamental unification.

References

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